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Contents

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PHOTOGRAPHIC RESOLVING POWER[1]

BY L. E. HOWLETT[2]

Abstract

The nature of photographic resolving power is discussed and attention called to the widespread misconception of it that exists at the present time. Most of the detailed discussion applies specifically to photographic objectives intended for use in aerial photography but the general approach to their photographic resolving power is applicable to studies of the photographic performance of all types of optical systems. An annulus type of target is proposed as more suitable than line targets. A method is given for the selection of the photographic focal plane when the essential requirement of the photographic use is the acquisition of maximum information. General remarks are made on the proper trend to be followed in the future design of photographic objectives. Results are presented on a study of a number of well known types of photographic objectives used for aerial photography.

Factors Affecting a Measure of Resolving Power

Ability to reveal information is one of the most important characteristics of an optical system. This applies equally to both visual and photographic systems. Accordingly it is not surprising that a great deal of importance has been attached to resolving power tests in making comparisons of the relative performance of two systems. Such tests have been used very successfully in so far as visual instruments are concerned, but it is unfortunate that for photographic objectives unsound procedures for carrying out resolving power measurements have materially restricted their usefulness. Most of them have been conducted with practically no appreciation of the fundamentals involved, and, as a result, are often worthless.

An optical system does not produce a perfect image of a point source. Instead, it forms an image in which the energy and frequency distribution depend on diffraction phenomena and residual aberrations. For good designs the perfection of the image on the axis of optical systems for visual use is ordinarily limited by diffraction phenomena. In photographic objectives, even of the best design, residual aberrations have so far limited the resolving power throughout the field of view. For either visual or photographic systems the image of a point source is symmetrical only on the axis. At other positions in the field the energy and frequency distribution are governed by the particular criteria used by the designer to obtain his corrections over the field

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covered by the lens. The lack of symmetry in the distribution of energy within the off-axis image is indicated by the astigmatism possessed to some degree by all objectives. If the design has been well executed by the optical shop it is reasonable to suppose that the distribution of energy in the image for any plane perpendicular to the optical axis is symmetrical about the radial direction, but a similar conclusion cannot be drawn with respect to the tangential direction.

The contrast and chromatic sensitivities of the energy sensitive device used for making observations on the image have a profound influence on a measurement made to determine the ability of a system to image as recognizable entities two closely positioned similar points. When visual resolving power measurements are made it is reasonable to suppose that the effect of residual chromatic aberrations on the measurement will be reduced to a certain extent by the filtering action of the visibility curve of the eye. The sharp peak of the latter will lead to some disregard of any red and blue light which may be responsible for the physical spread of the image. Many photographic emulsions have, compared with the eye, a reasonably uniform chromatic sensitivity over a considerable range. Accordingly when they are used in conjunction with the lens no corresponding disregard of chromatic aberrations can be expected. Transverse chromatic aberrations will be particularly important. The interpretation of an image made by a photoelectric cell will be different from that made by the eye. A thermocouple with its equal energy response will have still another one. Photographic emulsions will differ even among themselves.

Changes in the quality of the illumination, in the form or in the contrast of the test target will obviously lead to different distributions of energy in the image space. There will result different interpretations of the image by the same energy sensitive device, with consequent differences in the measures of resolving power.

A photographic objective currently considered of good quality strikingly shows the different interpretation made of residual aberrations by the eye and photographic emulsions by the way in which the resolving power varies with aperture for the two combinations. With the photographic emulsion the resolving power is increased by at least some reduction of aperture for lenses with apertures greater than $f/16$. The reverse is true when the eye is combined with the same lens. The facts are readily explained by supposing that, because of the difference between the chromatic and contrast sensitivities of the two means of observation, certain residual aberrations coming from the outer zones of the lens are disregarded by the eye but not by the photographic emulsion.

It would seem clear that, as a result of their preoccupation with the traditional visual tests of performance, designers of photographic objectives have thoughtlessly tested the correctness of their design procedures by these means. It is not therefore at all surprising that photographic objectives so designed have failed to utilize the full resolving power of photographic mate-

rials having sensitivity characteristics vastly different from those of the eye. Placing emphasis on performance testing by means of the energy sensitive device with which it is intended to combine the optical system does not require that all visual methods of testing be abandoned for optical instruments that are intended for combination with other energy sensitive devices. However, it does mean that such tests must be viewed with scepticism until laboratory experiments have shown in what way visual measurements can be correlated with the performance of the lens when combined with the sensitive receiver for which it is intended.

Since statements are so often made that imply that resolving power is an intrinsic property of an optical instrument it is excusable to recapitulate here the conclusions of the previous paragraphs. The factors governing the physical distribution of energy in the image space are:—

(1) The form and contrast of the test target,

(2) The quality of the light illuminating the target,

(3) The design of the optical system.

The factors governing the measure of resolving power are the three just mentioned and the chromatic and contrast sensitivities of the device used for making observations on the image.

It is obvious that stating the resolving power of an optical instrument as a certain number of lines per millimetre or the equivalent angle is a useful measure of the performance only if the factors listed in the previous paragraph are either implicitly understood or explicitly stated. It is further obvious that if the conditions of test involving these same factors do not closely approximate the conditions under which the lens is to be used in practice for the acquisition of information, the measure of resolving power obtained will have only an academic interest. It will be of no value in determining the degree of success attained by the designer in meeting the specific user requirements.

Visual Resolving Power Measurements

Resolving power measurements were probably first used to assess the value of astronomical lenses and other instruments intended for visual use. A general practice for making such resolving power measurements soon became well established. Line targets in which the lines are equal to the spaces between them are generally accepted as the target form. The contrast of the targets used is to all intents and purposes infinite. The ratio of the length of the lines to their separation has not been the same in all tests but sufficiently similar to make discrepancies negligible. Either natural daylight or tungsten illumination has been used. Minor differences between the measurements made with the two types of illumination may have occurred with certain optical systems but again it can be said that in general these differences have not been serious. The average eye has been the accepted means for making the observations in the image space. Since the optical systems were intended for visual use, the latter selection was an eminently logical one. The other

details of the testing procedure are a reasonably satisfactory conventionalization of the task generally imposed on optical instruments in visual use. Consequently visual resolving power tests conducted in different laboratories have been concordant to a useful degree and acceptable for judging the relative perfection of optical designs for visual uses.

Photographic Resolving Power

It is unfortunate that when the demand for photographic lenses occurred no steps were taken to reconsider the correctness of the hitherto accepted procedures for making resolving power tests. It is unlikely that those who were responsible for initiating the use of visual methods for judging the performance of photographic objectives would have considered for one moment that the testing of visual objectives with a thermocouple, photocell, etc., would provide useful information in regard to their visual performance unless a basis for correlating them with visual tests had been established experimentally. Failure to reconsider testing methods has resulted in the production of lenses that have good visual performance over a wide field of view but a rather indifferent photographic performance.

In proposing a correct method for making resolving power measurements on photographic objectives it is essential that one be prepared to take an entirely fresh approach. Long established methods for visual resolving power tests can be accepted only if after due consideration they are logical from the photographic point of view. In essence this means that thought must be given to two things. The first of these is that the objective is to be used photographically. The second is that different photographic uses may have different performance requirements and that specific consideration must be given to these in formulating a resolving power test that is a useful measure of the quality of the design.

In general the remarks made throughout this discussion primarily concern photographic objectives intended for aerial photography. The general approach however is the one that should be made to the problem of testing optical systems for any purpose.

For studying the performance of optical systems, each detail of the experimental procedure contributing to the various factors affecting a measure of resolving power must be established in such a way that the results of the test will give a measure of the designer's success in meeting the specific user requirements, and not some other ones of no interest to the user.

Gardner (2) was one of the first to emphasize the need for assessing photographically the performance of lenses intended for aerial photography. In his resolving power tests he substituted for the eye a high contrast, fine grain, slow photographic emulsion. By so doing he properly laid emphasis on the necessity of making resolving power tests on such lenses by photographic methods. There is however neglect of the fact that aerial photographic lenses are generally used with coarse grain, high speed materials, such as Aero Super XX. It is quite clear that the interpretation made of the energy distribution

in the image by the test emulsion and the emulsion actually used in most aerial operations will be vastly different. It therefore seems unlikely that the results of resolving power tests conducted with high contrast, fine grain, slow emulsions are of greater significance than those obtained visually or by any other means except Aero Super XX or its equivalent.

English Work on Photographic Resolving Power

An important contribution (1) to methods of measuring photographic resolving power was made during the war at the Kodak Research Laboratories, Harrow, England. Tearle of these laboratories conducted an extensive survey of the performance of available photographic lenses suitable for aerial photography. In this work two important proposals were made. The first was that the particular emulsion to be used in practice should be employed in the laboratory tests for assessing the performance of a photographic lens. The correctness of this needs no justification. The second proposal was that the high contrast test target hitherto traditional in resolving power measurements should be discarded in favour of a low contrast one on the basis that detail of interest in aerial photographs is generally of low contrast. The prevalence of low contrast detail in aerial photographs is hardly open to argument, and it seems reasonable that a low contrast target should be used in assessing aerial photographic objectives until it can be shown that, for all lenses, high contrast test targets yield more accurate corresponding results. A correspondence seems rather unlikely on qualitative arguments but extensive experimental results are required to clarify the situation. Tearle suggested the use of a log contrast of 0.2. It seems a reasonable one to adopt.

Consideration was also given in the Kodak Laboratories to the selection of a test target. This study had to be hurried since the problem of determining the relative value of different types of lenses for war purposes was extremely urgent. They selected the Cobb type. This consists of equal lines and spaces with the length of the lines three times the separation. Compared with other more generally used line targets, its smaller ratio of length to width of line favours the inclusion of the vagaries of emulsion blackening in the measure of resolving power. This is a desirable condition. At the time the Cobb target was selected the general resemblance of its form to city blocks was perhaps also attractive, since estimation of bomb damage was one of the principal requirements of aerial photography.

The Error of Attaching Undue Importance to Radial and Tangential Lines

Every effort was made in this laboratory to devise a form of target that would be more satisfactory for studying the performance of photographic systems than the conventional ones.

Tradition has attached great emphasis to resolving power on tangential and radial lines and it is perhaps an opportune time to call attention to the incorrect way in which radial and tangential resolving power are often designated in the literature. It is certainly correct to assume that resolving power is a

vector, and that radial and tangential resolving powers mean ability to resolve point sources in the radial and tangential directions respectively. If lines are substituted for points, radial resolving power must be measured on tangential lines and tangential resolving power on radial lines. The reverse definition is very prevalent in the literature and it seems very undesirable. To avoid either confusion or the use of an illogical expression it has been the custom in reports issued by the Optics Laboratory to employ the somewhat clumsy but unambiguous phrases 'resolving power on radial lines' and 'resolving power on tangential lines' when the terms 'radial resolving power' and 'tangential resolving power' might be misinterpreted. Some English workers have recently accepted this terminology. It is hoped that appreciation of the exact terminology will soon permit the use of the simpler expressions 'radial resolving power' and 'tangential resolving power' with the true sense.

The emphasis on resolving power in radial and tangential directions no doubt comes from the fact that the radial and tangential fields have been and still are of great interest to the designer in his computations. Photographically however there is no interest in either the radial or tangential fields, except in so far as their properties and relative positions may have a bearing on the resolving power in the photographic plane. The photographic plane must be selected according to a criterion that satisfies the user requirement. For some purposes the plane including the best photographic resolving power on the axis is the proper one. Other criteria will suit special purposes. Later, one will be proposed that takes cognizance of the requirements for aerial photographs. Whatever plane is selected it lies somewhere in the neighbourhood of the radial and tangential fields and the plane of best visual resolving power on the axis. The fact that there is no arithmetic procedure for using the resolving powers in the radial and tangential directions to derive resolving power in any other direction, or the mean resolving power with respect to direction, makes it difficult to justify the emphasis placed on resolving powers in these specific directions, unless the work for which the lens is intended is concerned only with resolving power in these two directions.

In aerial photographs the direction of interesting detail is random in the field of view. The interpretation of a photograph depends on the presence of a number of edges. The resolving power test must therefore concern itself with the disappearance of edges at the limit of resolving power. In aerial photographs the disappearance of an edge of limiting size is affected by edges in many other directions. Line targets are concerned with the disappearance of edges in the absence of edges in all other directions. Studies of this sort of resolving power performance by a lens in two specific directions is obviously not very useful in assessing its value for aerial photography. The mean resolving power on lines with respect to direction in the field is also inadequate, since no consideration is taken of the limitation that would be imposed on resolving power in a specific direction by edges in other directions. A target that will adequately measure resolving power of aerial objectives must take into account the mutually destructive effect of edges distributed at random.

The Proposed Annulus Target and Its Advantages

It is believed that a suitable conventionalization of the task that confronts a photographic lens can be made by using a target that is in the form of an annulus. In such a target the disappearance of the inner edge is affected by an infinitely small neighbouring edge in every other direction. The target therefore places no emphasis on resolving power ability in any special direction of the photographic plane. As the limit of resolving power of a lens is approached, some distortion of the form of detail inevitably occurs. In order that the recording of the detail can provide useful information, the distortion must not be sufficiently great to prevent recognition. Cognizance of recognizability is therefore very desirable in establishing a criterion for the limit of resolving power. Use of the annulus type target permits the introduction of the recognizability factor into the criterion for determining the limit of resolving power. The limit of resolving power can be considered as the smallest target that is recorded on the test negative as a complete boundary, even though the boundary is distorted in shape. Conversely, resolution is not attained if the boundary is broken or if it ceases to be a boundary by reason of the disappearance of the central area. Thus it is accepted that a certain amount of distortion is permissible without loss of recognizability. The arbitrary amount introduced by the above criterion is considered reasonable.

In work undertaken in this laboratory that involves the use of an annulus target, a light annulus has been used on a darker background. The diameter of the central portion has been one-third the total diameter of the annulus. In setting a numerical value to the limit of resolving power the annulus has been treated as the equivalent of lines having a separation equal to the diameter of the central portion of the annulus. This is an arbitrary procedure that serves well enough but it is very possible that there is a better way of assigning a numerical value.

Specific tests to assess the utility of the annulus type of target and the very considerable use made of it in varied experimental work have shown it to be a rather satisfactory one. Test strips are easy to read. In fact, it has been possible to employ usefully targets in which the size ratio is the sixth root of two. The labour of reading the targets and computing results is less than that with line targets, since one set of results or curves replaces the two required at every step of the procedure with radial and tangential lines. It allows the position of best focus to be selected at any position of the field without ambiguity and without the necessity of making this decision on the basis of either a subjective compromise between radial and tangential resolving power or the introduction of an illogical arbitrary arithmetic method of averaging.

A Method of Selecting the Photographic Focal Plane

It has already been pointed out that the photographic plane must be selected by means of a criterion that takes full cognizance of the specific purpose of the photographic operation. Visual instruments are usually focused so that maximum resolving power is attained on the axis. Any curvature of the

field is compensated to a certain extent by the adaptation of the eye. No such advantage is present photographically, although the points of best photographic resolving power for various angular positions in the field also lie not on a plane but on a curved surface. It would be very satisfactory if the focal plane of a camera could be made to fit this curve, but in general this is not practical. Existing cameras provide flat focal planes, and accordingly it is necessary for the purposes of focusing to select a plane perpendicular to the optical axis of a lens that has a photographic performance that is most suited to the photographic requirements. For the great bulk of amateur and commercial photography the plane having the best photographic imagery on the axis will probably be chosen since there is a certain advantage in being able to emphasize the object of maximum interest by its sharpness, and to reduce interest in the less important neighbouring objects by having them less well defined. In aerial photography the requirement is quite different. Whether the purpose is military reconnaissance or aerial survey it is essential that the maximum possible amount of information be obtained from a photograph. It seems reasonable to suppose that equal importance should be attached to all detail irrespective of its size or position in the field of view. If this assumption is accepted it follows that we should choose as the focal plane for aerial photography the one that has the best average photographic resolving power with respect to area over the picture area or the maximum value of $\frac{1}{A}\int_0^A RdA$, where R is resolving power and A is the negative area.

This plane can be determined by plotting a curve relating average resolving power with respect to position along the optical axis of planes perpendicular to the optical axis. The performance of the lens in respect of maximum average resolving power can be accepted as an important measure of its usefulness for aerial photography. Comparison of lenses on this basis might prove troublesome were it not for the fact that aerial cameras are provided with a limited number of picture sizes. Only two sizes have any degree of general popularity in the English-speaking world. These are a 9 by 9 in. format and a 5 by 5 in. one. Of these, the former is much more common for serious work and will probably become a general standard for some time to come. Accordingly, it is logical that this area be chosen for averaging resolving power. Designers will tend to work to this area. In fact, if they did not their lenses would be of little interest in aerial photographic operations unless the field of view, focal length, and aperture, combined with the average resolving power, were sufficiently unusual to justify a manufacturer's embarking on production of a camera to suit their picture size. Consideration also has to be taken of the large amount of existing photogrammetric equipment particularly suited to the 9 by 9 in. size.

It is not necessary that a complete series of photographic resolving power tests for a number of planes and apertures be carried out for the focusing of every camera. Once a lens type has been thoroughly studied, the separation of the plane of best average resolving power from the position of best visual

resolving power on the axis can be determined and cameras focused visually by means of it. In this connection however it should be pointed out that since the photographic plane selected on the criterion given here does not in general coincide with position of best visual focus, cameras intended for survey purposes should have their geometrical calibration carried out by photographic methods. Gardner (2) has already emphasized the desirability of such a procedure.

Experimental Equipment and Procedure

In 1942, studies of the photographic performance of optical systems were begun in the Optics Laboratory of the National Research Council of Canada. Owing to the exigencies of war requirements, attention was mostly paid to lenses that might be suitable for aerial photographic reconnaissance. The problem of assessing for this purpose the relative value of the various lenses available was of the highest importance.

The original apparatus was practically identical with that (1) used by the Kodak Research Laboratories, Harrow, England, but very considerable changes in the mechanical design and construction were soon found to be desirable in order to provide rigidity, convenience of operation, and reproducibility of mechanical settings. The latest version of the equipment permits the use of either a 30 ft. collimator with a 10 in. aperture or a 36 in. collimator with a 3 in. aperture. Space is conserved when the 30 ft. collimator is in use by folding the beam three times by means of three aluminized 10 in. optical flats. The support for the collimator, the testing bench, and auxiliary equipment is a massive welded angle iron structure insulated from the floor by Lord shock absorbers. These have a natural frequency of 100 cycles per min. and quite successfully protect the testing equipment from the high frequency vibrations of the building. In front of the collimator is the testing bench made from a magnesium alloy casting approximately 60 in. long. This supports the lens under study and the film holder. The testing bench can be rotated on a vertical bearing situated at one end. Rollers on the underside of the other end rest on a rail that is curved to form part of the circumference of a circle having a radius slightly shorter than the casting. A clicker device allows the testing bench to be reliably positioned on the rail in the dark so that its axis makes a definite angle with the optical axis of the collimator. The positions are at every $2\frac{1}{2}°$ up to 15° off-axis and at every 5° from there to 45° off the axis. The automatic positioning is a valuable convenience, since the room housing the apparatus is in effect the camera of the instrument. The holder for the lens is movable along the axis of the testing bench, so that any lens can be so positioned with respect to the vertical axis of rotation that at all angular positions of the testing bench off-axis the lens is completely flooded with light from the collimator. The film holder can be slid manually on a dovetail the whole length of the testing bench. This arrangement permits the plane in which the film is held to be brought quickly to the region of the image space where the study will be made. A second dovetail equipped

with a micrometer screw on top of the first dovetail enables the film holder to be moved along the optical axis of the test lens in predetermined, accurate, and reproducible steps. The film can be held in a plane perpendicular to the optical axis of the lens either by a suction back or a register glass, depending for which method of defining the focal plane the lens was designed. The resolving power target is placed in the focal plane of the collimator and illuminated by an integrating sphere. The uniform illumination so provided is equivalent to mean noon sunlight modified by a Wratten No. 12 filter. This filter is included because it has come into general use for ordinary daylight aerial photography. When the purposes for which the lens is intended involve a different quality of illumination the appropriate changes are made to provide the same quality in the apparatus. Exposures of the target are made by means of an electric timer that turns the light of the integrating sphere on and off.

In setting up a lens for study, autocollimation is used to make the film plane parallel within a minute of arc to the machined surface of the mount which is used as a reference plane when mounting the lens in a camera. Since the making of this plane and the focal plane parallel in the camera is the best treatment that the lens manufacturer can expect, it seems quite fair to judge the performance of his product under these geometrical conditions.

The film holder accommodates 14 in. strips of 35 mm. film. Exposures are made at all marked lens apertures and at specific angles in the field for a number of planes perpendicular to the optical axis in the neighbourhood of the point of best visual focus on the axis. For narrow angle lenses exposures are made every $2\frac{1}{2}°$ in the field and for wide angle lenses every 5°. The exposure is constant at all positions in the field for the same aperture and is chosen so that the density recorded on the test strips is that required to give the maximum resolving power of the photographic material. When a lens possesses such excessive vignetting that a constant exposure does not give the optimum density at all angular settings, the exposure is chosen that yields the optimum density for the angular setting that represents the largest part of the negative area for which the lens is designed. Use of constant exposure is at variance with the practice of English workers (1), but it reproduces more realistically the conditions under which the lens must operate in a camera than use of the several exposures required to give optimum density at all angular settings.

The resolving power targets are made on Eastman HR emulsion. Methods that have been developed for the convenient production of these to rigid specifications of geometric perfection and contrast will be described elsewhere. In the early work the Cobb target adopted by the Kodak Research Laboratories, Harrow, England, for similar studies was employed but it was later discarded in favour of the annulus target. Most of the numerical data presented here were derived by the use of the annulus target. The exceptions are duly noted. In these cases the data were acquired before the annulus target was adopted. Because the lenses concerned were no longer available the experiments could not be repeated with the annulus target. Use of detailed results

based on the Cobb target has been restricted to a minimum because they are not strictly comparable with those obtained by the annulus target. With the majority of lenses the latter target indicates a more rapid deterioration of resolving power off-axis than line targets. The amount of the difference is a function of the distribution of energy in the off-axis image. The reason for its occurrence is obvious. Line targets measure resolving power on edges in a specific direction without interference by edges in other directions. The annulus target measures the mutually destructive effect at the limit of resolving power of edges in all directions. Since the latter condition is certainly more representative of actual conditions in the photographic plane during use than those implied by line targets, it seems reasonable to suppose that the results obtained with the annulus target are more useful in evaluating performance.

For a lens with four marked apertures studied in seven planes, 28 strips of film are required to make a complete survey of the photographic performance. Aero Super XX emulsion is used and processed under constant conditions in D-19b to a gamma value of 1.35. The particular emulsion and processing conditions are chosen because they are the ones that actually obtain in the majority of aerial photographic operations carried out in Canada and elsewhere.

The resolving power strips are read with the aid of a low power binocular microscope with the conditions of magnification and illumination so adjusted that the reading of maximum resolving power is favoured.

The data are first plotted as through-the-focus curves for each angle in the field of view. From these curves values are obtained for plotting the off-axis resolving power curves for the several planes studied experimentally. This procedure tends to smooth out the erratic measures of resolving power that periodically occur for one reason or another and seem practically impossible to prevent. It is considered to be more useful in all curves representing the performance of aerial photographic lenses to plot the logarithm of the ground resolving power instead of the resolving power itself. The ground resolving power is defined as RF, where R is the resolving power in lines per millimetre and F is the focal length expressed in millimetres. It has generally been the custom to plot off-axis curves with the abscissae directly proportional to the angular position represented. Instead of following this method in presenting the results given here, the abscissae are designated by the angles they represent but they are made proportional to the negative area out to that position in the field. The reason for the crowding of the abscissae near the origin is obvious and the similar compression at greater angles in the field is due to the square format of the picture. Curves plotted in the latter way are superior to those plotted in the customary manner because the average resolving power on an areal basis, which is so important in judging relative performance of aerial photographic lenses, is immediately appreciated. The average ordinate of the curve is the average resolving power of the area over which the ordinate is averaged. Average resolving powers given for the various lenses

reported here are for a 9 by 9 in. format. This is chosen because it is the most common one for aerial operations.

From a curve relating the logarithm of average resolving power to position along the optical axis the plane of best average resolving power can be selected as the photographic plane to which the camera should be focused. The displacement, if any, of the plane of best average resolving power from the position of best visual resolving power on the axis is measured. This information permits the camera to be focused correctly for photographic use by employing a simple visual criterion.

Precision of the Experimental Results

It is a matter of some importance to know the degree of accuracy that it is possible to attain in photographic resolving power measurements. In the Optics Laboratory this point has been studied using two lenses of different types. A complete series of tests were carried out five times with each lens. Each test included the complete setting up of the lens in the testing equipment. By these experiments it has been shown that the maximum error likely to occur in any individual measurement of average photographic resolving power is $\pm 7\frac{1}{2}\%$.

Experimental Results for a Number of Lenses

The performances of a large number of photographic objectives intended for aerial photography have been studied in the Optics Laboratory by following the techniques described. A graphical presentation of the results obtained with representative samples has been made in the adjoining pages. Lenses varying widely in focal length and field of view are included.

For convenience, a summary follows of the more important conditions that obtained in the experiments. Where in a few cases departures have been made from these conditions the fact is duly noted on the relevant graphs.

1. Collimator for placing the target at infinity.
2. Target of annulus type having a log contrast of 0.2 and the $\sqrt[6]{2}$ as the size ratio.
3. Target illuminated by an integrating sphere.
4. Quality of illumination is mean noon sunlight modified by a Wratten No. 12 filter.
5. Recording done on Aero Super XX emulsion.
6. Constant exposure across the field.
7. Exposure chosen to give optimum density for maximum resolving power of the photographic material over the largest possible area of the field covered by the lens.
8. Development in D-19b to a constant gamma value of 1.35.

A large number of graphs is required to present all the information acquired in a complete study of one lens. To avoid the inclusion here of a multiplicity

of graphs there is given in general for each lens only the following, which are sufficient to give the final over-all performance.

1. The curve relating the logarithm of the average ground resolving power (Log $\overline{RF}$) over a 9 by 9 in. picture area to position along the optical axis.
2. Off-axis curves relating the logarithm of the ground resolving power (Log RF) to the angular position in the field for the several planes in which measurements were made. The axial position is noted on each graph. Values increase for planes further from the lens.

Additional graphs are presented to illustrate special points.

In considering the results it should be recalled that the resolving power of Aero Super XX emulsion as determined by contact printing with a test target having the same form and contrast as that used in the experiments is of the order of 22 lines per millimetre. This is substantially in excess of the resolving power obtained anywhere in the field when photographic lenses are used in conjunction with the emulsion.

Figs. 1 to 6 show the performances of lenses that have been used or proposed for high altitude military reconnaissance. Such reconnaissance is conducted at altitudes in excess of 20,000 ft.

Fig. 1 shows the performance of a typical Booth 36 in. $f/6.3$ telephoto lens. Lenses of this design were standard in the Royal Air Force and the Royal Canadian Air Force for high altitude photography. The particular one reported is representative of some ninety that were made in the Optical Shop of the National Research Laboratories for the Ministry of Aircraft Production of the United Kingdom at a time when the supply of them from ordinary sources was limited. The Booth telephoto was probably the best lens of long focal length in actual operational use for high altitude reconnaissance by the United Nations during World War II.

Fig. 2 shows the performance of a Bausch & Lomb 40 in. $f/8$ telephoto. This lens is inferior in performance to the other members of the group represented by Figs. 1 to 6.

Fig. 3 shows the performance of a captured German Zeiss Telekon 75 cm. $f/6.3$ lens. This lens has unusually good photographic performance. It is definitely superior to the Booth telephoto, having a slightly larger field of view and better ground resolving power. The photographic resolving power is maintained much better off-axis. In making these statements however it should be mentioned that a recent redesign of the Booth, which is not yet in production, is said to have improved the performance to a point where it may be considered the equal of the Zeiss Telekon.

Figs. 4, 5, and 6 give results on the Eastman 24 in. $f/8$ Aerostigmat, the Eastman 24 in. $f/5.6$ Aero Ektar, and the Ross 20 in. $f/6.3$ Xpres. These three lenses are of special interest. They are prototypes constructed to designs made specifically with a view to improving photographic performance. The effort was notably successful. In all lenses the photographic resolving power is well maintained off axis. The two Eastman lenses are somewhat

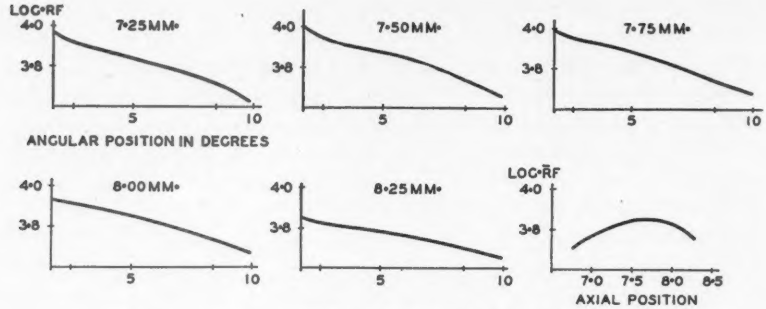

FIG. 1. *Performance of Booth 36 in. f/6.3 telephoto at f/6.3.*

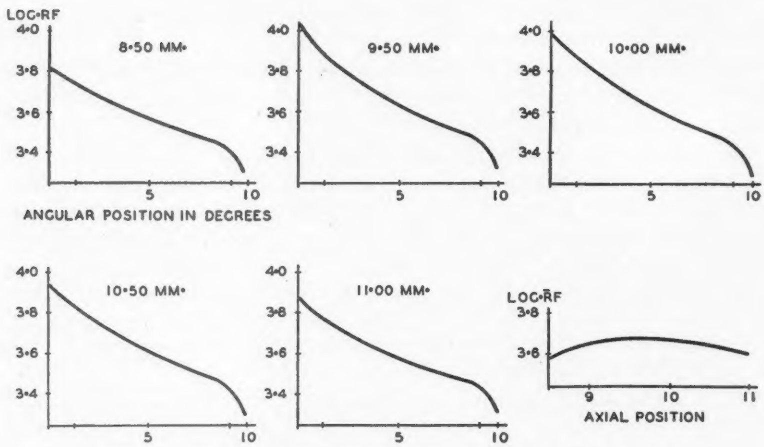

FIG. 2. *Performance of Bausch & Lomb 40 in. f/8 telephoto at f/8.*

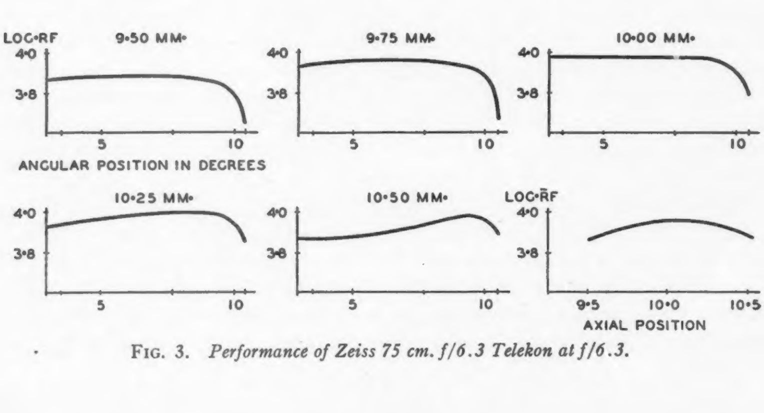

FIG. 3. *Performance of Zeiss 75 cm. f/6.3 Telekon at f/6.3.*

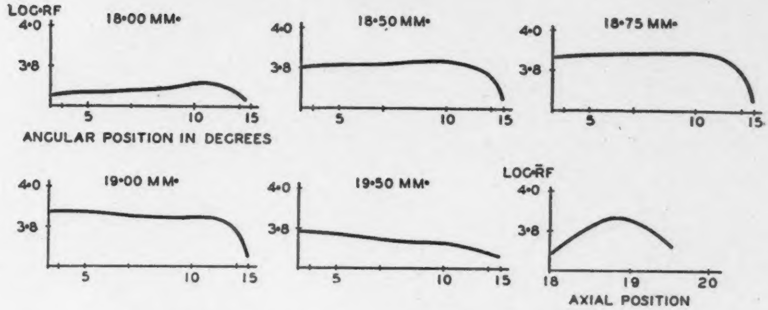

FIG. 4. *Performance of Eastman 24 in. f/8 Aerostigmat at f/8.*

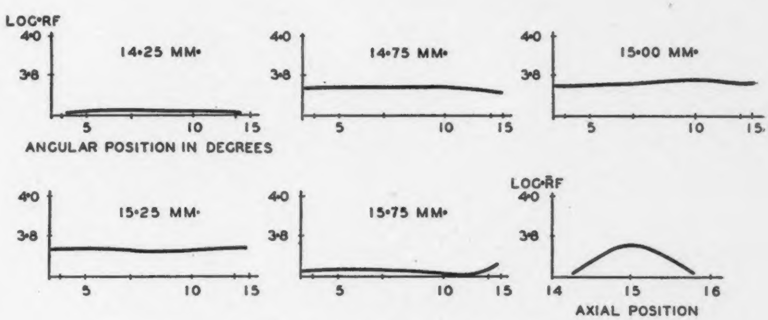

FIG. 5. *Performance of Eastman 24 in. f/5.6 Aero Ektar at f/5.6.*

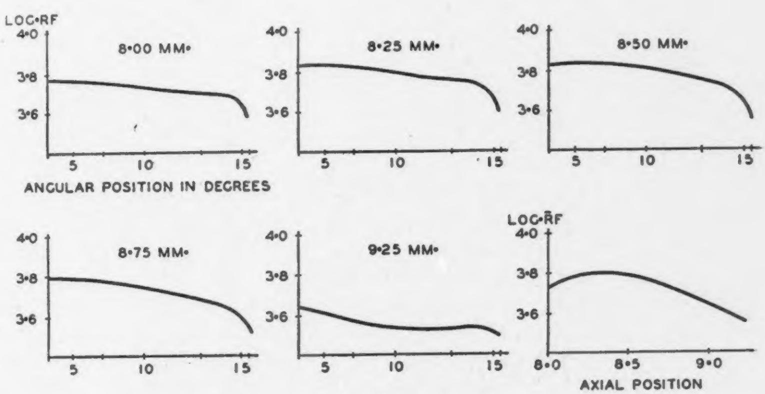

FIG. 6. *Performance of Ross 20 in. f/6.3 Xpres at f/6.3.*

superior in this respect. They have the same average resolving power as the Booth telephoto, with a considerable increase in the field covered on a 9 by 9 in. negative. The Ross has not quite the ground resolving power of the Booth but its design is much superior, considering that it covers an even larger angular field of view on a 9 by 9 in. picture than the two Eastman lenses. It can be remarked that the complexity of design and construction of the Ross lens appears to be considerably less than that of the other two. It is also lighter and smaller. It is said that the Ross company have improved the performance by redesign, but as yet the prototype has not been available to this laboratory for study.

Figs. 7 and 8 show the performances of the Ross 10 in. $f/4$ Xpres and the Eastman 14 in. $f/6.3$ Anastigmat Ektar. They are typical of objectives for aerial photography at moderate altitudes. They are good lenses by all standards except that of the lenses shown in Figs. 3, 4, 5, and 6.

Figs. 9 and 10 show the performances of the Ross 6 in. $f/5.5$ and the Bausch & Lomb 6 in. $f/6.3$ Metrogon. These are very wide angle lenses for aerial survey use and are of comparable quality.

Fig. 11 shows for the Zeiss 75 cm. $f/6.3$ Telekon, the Eastman 24 in. $f/5.6$ Aero Ektar, and the Ross 6 in. $f/5.5$ wide angle lens, the logarithm of the average resolving power for different apertures plotted against aperture to illustrate the characteristic increase in average resolving power that occurs with all photographic objectives when the aperture is reduced.

Fig. 12 shows the uniformity of performance that can be expected in several photographic objectives of the same type. Four Booth 36 in. $f/5.3$ telephoto lenses are included and 10 Bausch & Lomb 6 in. $f/6.3$ Metrogons. In appreciating these graphs it should be recalled that the error to be expected in measurements of photographic resolving power is $\pm 7\frac{1}{2}\%$.

When the original studies were made on the Bausch & Lomb $f/8$ telephoto lens, the separation of front and rear components recommended by the manufacturer was used in the laboratory experiments. The results seemed to indicate that at $f/8$, its maximum aperture, the Bausch & Lomb lens was barely the equivalent of the Booth 36 in. $f/6.3$ telephoto at its maximum aperture of $f/6.3$. Recognizing that telephoto lenses are notoriously sensitive to the separation of the front and rear components, it was thought of interest to determine whether the performance of the Bausch & Lomb lens could be improved by a change in separation.

Fig. 13 shows the results of the experiments. The logarithms of average ground resolving powers for the planes of best average resolving power for a number of separations are plotted against separation. The variation of average resolving power with axial position for the best separation is also shown. The investigation was carried out at the time when the transition was taking place from the use of the Cobb target to the use of the annulus target. Consequently results were obtained with both types and are presented. The results of the parallel investigations are of particular interest since they show the greater usefulness of the annulus target for the solution of such problems. The best

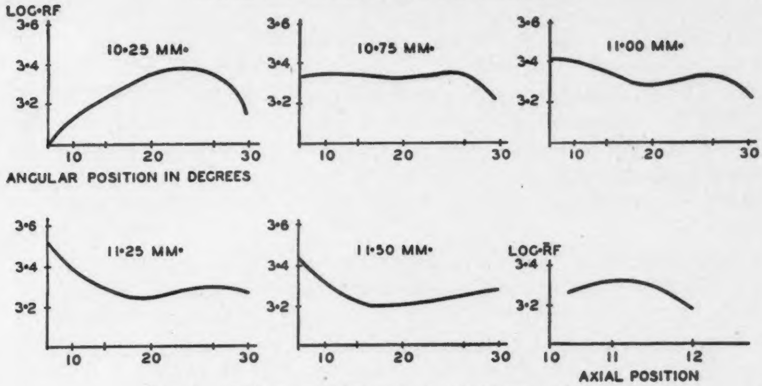

FIG. 7. *Performance of Ross 10 in. f/4 Xpres at f/4.*

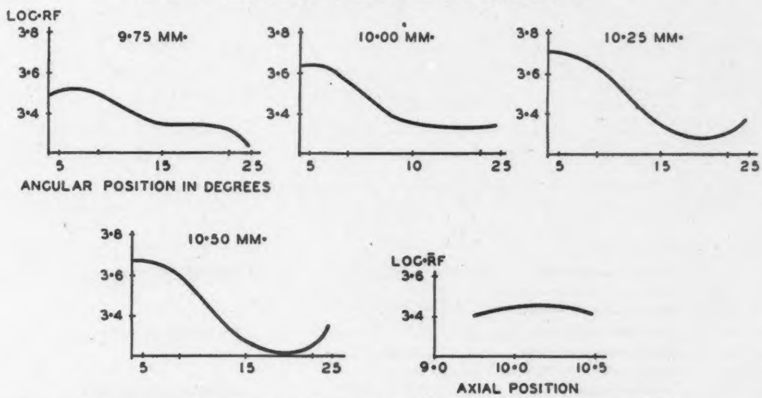

FIG. 8. *Performance of Eastman 14 in. f/6.3 Anastigmat Ektar at f/6.3.*

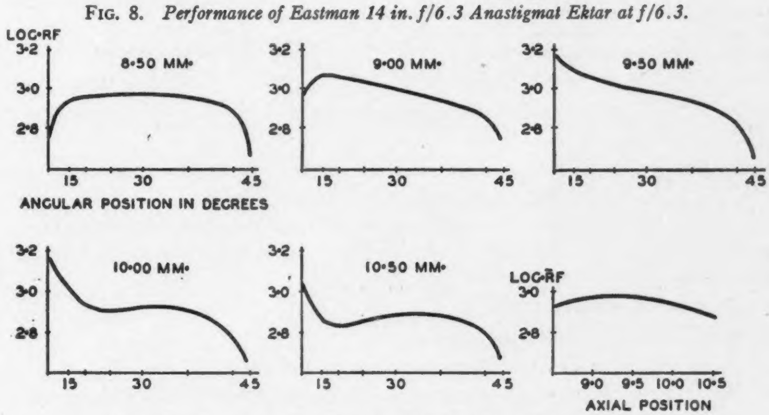

FIG. 9. *Performance of Ross 6 in. f/5.5 wide angle lens at f/5.5.*

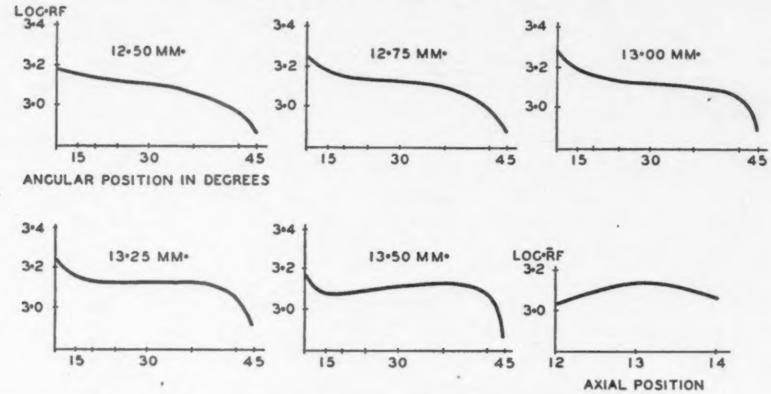

FIG. 10. *Performance of Bausch & Lomb 6 in. f/6.3 Metrogon at f/6.3.*

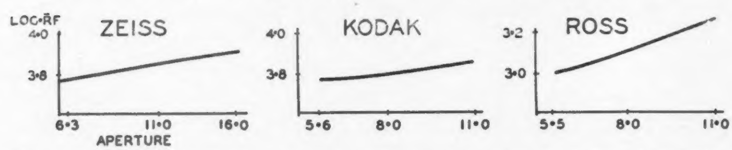

FIG. 11. *The variation of average resolving power with aperture for a Zeiss 75 cm. f/6.3 Telekon, an Eastman 24 in. f/5.6 Aero Ektar, and a Ross 6 in. f/5.5 wide angle lens.*

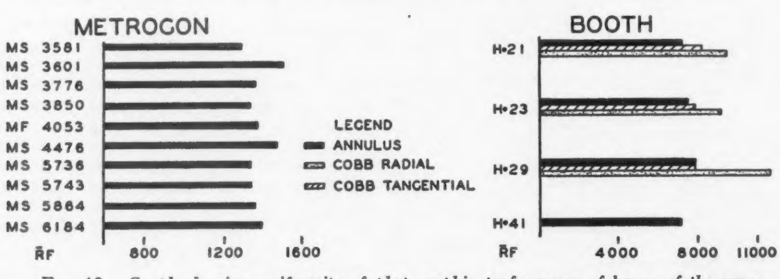

FIG. 12. *Graph showing uniformity of photographic performance of lenses of the same type.*

separation can be unambiguously selected on the basis of the annulus target at 8.200 in. The setting of the separation as received was 8.240 in.

It would be something of a problem to select the correct separation from results given in Fig. 13 for the Cobb target. The following various bases of selection would be competitive.

(1) Best radial resolving power.

(2) Best tangential resolving power.

(3) A subjective selection from inspection of the curves.

(4) An arbitrary mathematical method of averaging the radial and tangential resolving powers.

The annulus target indicates that the best separation is that for which the lower resolving power (that on tangential lines) reaches its maximum and not at the position giving maximum resolving power on radial lines. The latter seems to have been used by the manufacturer as the criterion for selecting the separation. The experimental result confirms the subjective impression already formed in the laboratory that with line targets the lesser of the two resolving powers is generally the better rough, but not necessarily reliable, guide in assessing performance.

Similar experiments were conducted to determine the best separation for the Booth 36 in. $f/6.3$ telephoto. These showed that the best separation determined photographically is the same as would be selected by a visual criterion requiring that the spherical aberration be zero for the 0.65 to 0.75 zone of the lens. It is understood that previous to our experiments this visual criterion was adopted by the Ross company for setting the separation of Booth telephotos produced in their factory.

Fig. 14 shows for both annulus and Cobb targets logarithms of average resolving power plotted against axial position for the Booth lens at its optimum separation. Consideration of Fig. 14 in conjunction with Fig. 13 shows a difference in the relative assessment of the two lenses made by the two targets. If the separation indicated by the results with the annulus test target

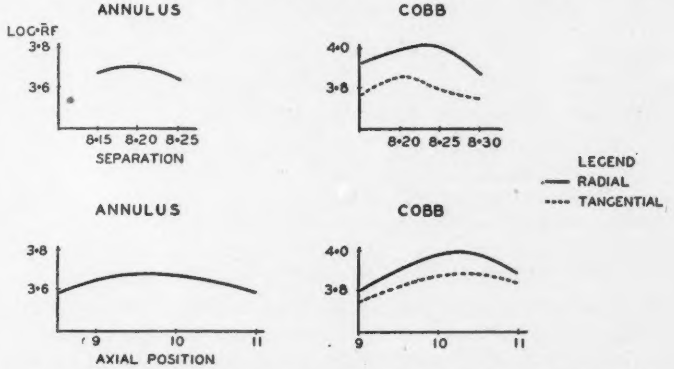

FIG. 13. *Curves for establishing the proper separation for Bausch & Lomb 40 in. f/8 telephoto and average resolving power curves for the best separation plotted against axial position.*

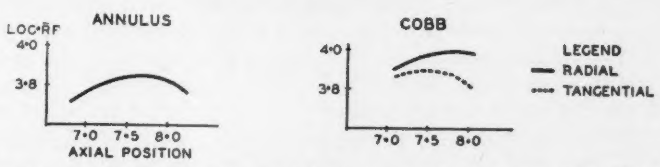

FIG. 14. *Curve showing average resolving powers for Booth 36 in. f/6.3 telephoto at f/6.3, on the basis of both the annulus and the Cobb target.*

in Fig. 13 is selected for the Bausch & Lomb lens it will be noted that the logarithm of the average resolving power on tangential lines is approximately the same as for the Booth, and that the logarithm of the average resolving power on radial lines for the Bausch & Lomb lens is somewhat greater. This suggests that at $f/8$ the Bausch & Lomb lens is comparable with the Booth lens at $f/6.3$. However, the logarithm of the average ground resolving power for the annulus target is 3.74 for the Bausch & Lomb lens and 3.86 for the Booth lens. Thus allowing for probable experimental error the annulus target indicates that the Booth lens is significantly superior at $f/6.3$ to the Bausch & Lomb lens at $f/8$. It is felt that arguments made earlier for the use of the annulus target justify the conclusion that results obtained by it lead to a more correct evaluation of the relative performance of the two lenses than results from line targets.

The deficiency in performance shown by the particular sample of the Bausch & Lomb 40 in. $f/8$ telephoto used in the tests may not necessarily be characteristic of the whole class. A production mistake may have been responsible. Nevertheless service experience tends to support the conclusion that the Booth is the better lens.

The Eastman 24 in. $f/5.6$ Aero Ektar was designed for the suction back method of holding the film in the focal plane. Fig. 15 shows that it is immaterial from the point of view of lens–film resolving power on Aero Super XX whether this method is used or a $\frac{1}{8}$ in. register glass.

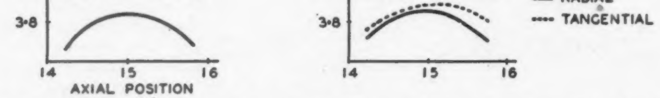

FIG. 15. *Average resolving power curves for Eastman 24 in. $f/5.6$ Aero Ektar at $f/5.6$, with a suction back and with a 1/8 in. register glass.*

Comparison of High and Low Contrast Targets

For three lenses, the 24 in. $f/5.6$ Aero Ektar, the Ross 20 in. $f/6.3$ Xpres, and the Bausch & Lomb 40 in. $f/8$ Telestigmat, high contrast results have been obtained with the annulus and Cobb targets in addition to low contrast targets of the same types. There are too few to draw general conclusions but because they are at least suggestive of interesting ones they are presented in Table I. The numbers in the columns are logarithms of average ground resolving power over a 9 by 9 in. field. In the case of the Cobb target these logarithms have been given for radial lines, tangential lines, and the mean for radial and tangential lines because of the intrinsic uncertainty as to which value is the best guide to performance.

The high contrast results with the Cobb target indicate that the lenses are very comparable. The low contrast results with the Cobb target give the same view. The annulus target on both high and low contrast makes Aero Ektar

TABLE I

COMPARATIVE RESULTS ON THREE LENSES WITH BOTH ANNULUS AND COBB TARGETS AT HIGH AND LOW CONTRAST

$\bar{R}$ = Average resolving power on radial lines over 9 by 9 in.

$\bar{T}$ = Average resolving power on tangential lines over 9 by 9 in.

$\frac{\bar{R}+\bar{T}}{2}$ = Arithmetic mean of average resolving powers on radial and tangential lines over 9 by 9 in.

Numbers in the columns are logarithms of the average resolving powers indicated.

Lens	High contrast				Low contrast			
	Annulus	Cobb			Annulus	Cobb		
		$\bar{R}$	$\bar{T}$	$\frac{\bar{R}+\bar{T}}{2}$		$\bar{R}$	$\bar{T}$	$\frac{\bar{R}+\bar{T}}{2}$
Aero Ektar	3.90	4.08	4.08	4.08	3.76	3.86	3.86	3.86
Ross Xpres	3.96	4.02	4.02	4.02	3.81	3.86	3.86	3.86
Bausch & Lomb Telestigmat	3.84	4.17	4.00	4.09	3.70	4.00	3.84	3.92

and the Xpres very comparable in performance, with a slight preference for the Ross. Both the high and the low contrast annulus make the Bausch & Lomb Telestigmat definitely inferior.

Correlation of Laboratory Results with Actual Practice

In order to find the degree of correlation between performance measurements made in the laboratory on aerial photographic objectives by these techniques and actual performance under operational conditions, a ground resolving power target was constructed at Rockcliffe Air Station. The target was a flat slab of cement with its surface suitably roughened to obtain a good approximation of diffuse reflectivity. A target having the same form and contrast as the target used in the laboratory experiments was painted on it. For supporting the cameras in the aircraft use was made of the simple and efficient form of mount developed by J. B. Reid (1), which employs sponge rubber for damping. In the various flight trials the exposures were so selected that during an exposure the image movement resulting from the speed of the aircraft was of negligible importance in limiting resolving power. In a series of tests with lenses of widely different focal lengths and apertures it was found that the ground resolving power measured in laboratory experiments was attained in practice.

Degree of Accuracy Required in Lens Making

The failure of lens–film combinations to utilize, under aerial photographic conditions in either the air or the laboratory, the full resolving power of the photographic material tends to suggest a question as to whether the degree of suitability attained by current photographic designs for the emulsions used

in practice justifies the meticulously careful working of the design in the optical shop. That quite small departures by the optical shop from a designer's specification seriously reduce the visual performance of a photographic objective is to be expected, since in the design much weight has been given directly or indirectly to the sensitivity characteristics of the eye. It would be small justification of the difficult computations involved if the prescription could be carelessly filled with no loss of visual performance. It by no means follows, however, that such departures from the prescription are equally important when the lens is to be used in combination with an emulsion having vastly different sensitivity characteristics, and in which so much lower resolutions are involved. It is planned to investigate the point experimentally in this laboratory.

A Graphical Presentation for Comparative Purposes

The information given by the results of laboratory experiments reported here is adequate for selecting lenses for any aerial photographic operation. It is true that distortion characteristics are important in so far as aerial surveys are concerned, but, since for that purpose only lenses that have a sufficiently low maximum distortion need be considered, this particular characteristic can be omitted from the following discussion, which is directed towards developing a graphical presentation of the essential information on lenses so that a quick comparison of them for aerial photography can be made.

Emphasis will be placed on making the proper compromise between the factors affecting the amount of information that can be gained per picture. These factors will be partly governed by the operational requirement and partly by design limitations. They are:—

1. Ground resolving power,
2. Field of view,
3. Aperture,
4. Focal length.

It is reasonable to suppose that designers of similar ability with the same glasses at their disposal and working to similar criteria will produce designs leading to reasonably equivalent ground resolving powers for the same field of view, maximum aperture, and focal length. Marked improvement of design can result only from the availability of a more useful glass or from a change in design methods. Consequently in any multidimensional graphical representation of the performance and optical dimensions of a number of lenses, it would be expected that those of equally successful design would lie on approximately the same surface. The general correctness of this almost axiomatic statement has been shown by the actual compilation (1) of such graphs from experimental data. It is doubtful that such presentations yield useful theoretical information or assist designers in other ways. They are certainly not a convenient means for solving the practical problem of selecting a lens having the best design with the optical dimensional requirements imposed by a specific aerial photographic use.

For aerial photography and no doubt for a number of other user requirements, the working conditions permit a reduction of the number of variables that require to be treated individually in a graphical presentation. Recognition that it is desirable for aerial photographic lenses to cover an acceptable standard format can be used to advantage. It has been already pointed out that at present the 9 by 9 in. size is more nearly standard in aerial photography than any other. Acceptance of it or acceptance in the future of a more logical circular format permits the focal length to be omitted explicitly from the presentation, because it is implicitly included with the angular field of view. The variables to be related are therefore reduced to three.

For each maximum aperture the ground resolving power can be plotted as a function of the field of view on a two-dimensional graph. Lenses of the same aperture and of equivalent design will fall on a smooth curve. Those of superior design will fall above the curve, and those of inferior design below.

Figs. 16 and 17 show, for a number of lenses, the dimensional characteristics and performances, based on the Cobb and annulus targets, respectively, plotted in the way just described. The abscissa is the logarithm of the square of twice the tangent of the half-angle of view on a 9 by 9 in. picture area. The ordinate is the logarithm of the average ground resolving power over the same area. The majority of the lenses shown in the two figures have a maximum aperture of $f/6.3$. A few other apertures of some of the lenses and a few lenses having different maximum apertures have been included as a matter of interest, so that their characteristics can be judged in comparison with those having a maximum aperture of $f/6.3$. The numbers by the points on the graph identify the lenses. If the lens has a maximum aperture different from $f/6.3$ or was tested at another aperture, the fact is recorded.

Results are available on fewer lenses than would be desirable but there are sufficient of them to show the practical usefulness of the mode of presentation and the desirability of making it more complete. On both graphs, lines have been drawn that suggest the curve on which lenses of maximum aperture $f/6.3$ should fall if they possess equivalent perfection of design. In effect, lenses on this curve are designed to give on Aero Super XX irrespective of focal length the same photographic size to the image of a point source. More points representing lenses would be required to locate definitely the curve. Pending acquisition of these however a straight line has been drawn to illustrate tentatively the usefulness of this mode of presentation. It is unlikely that the actual function is a straight line. It would seem probable that some departure from a straight line would occur for lenses of larger diameters, because of the difficulty of obtaining high quality glass blanks for such lenses. A second straight line has been drawn on each figure to show the standard of improved quality that is suggested by the Telekon and the Ross Xpres for lenses of maximum aperture $f/6.3$. The Aero Ektar and the Aerostigmat are a considerable improvement on existing types but do not quite equal the design quality of the Telekon and the Ross. The Bausch & Lomb Telestigmat although having a maximum aperture of only $f/8$ falls considerably below the

standard set by the general run of lenses at $f/6.3$. The performance of a number of the lenses at smaller apertures is also shown on both graphs. Improvement on stopping down is less than the improvement in design accomplished in the Telekon and Ross Xpres as compared with the previous standard.

Figs. 16 and 17 are useful in two ways. The relative quality of design can be assessed for use with the 9 by 9 in. picture size. The necessary operational compromise between ground resolving power and field of view can be con-

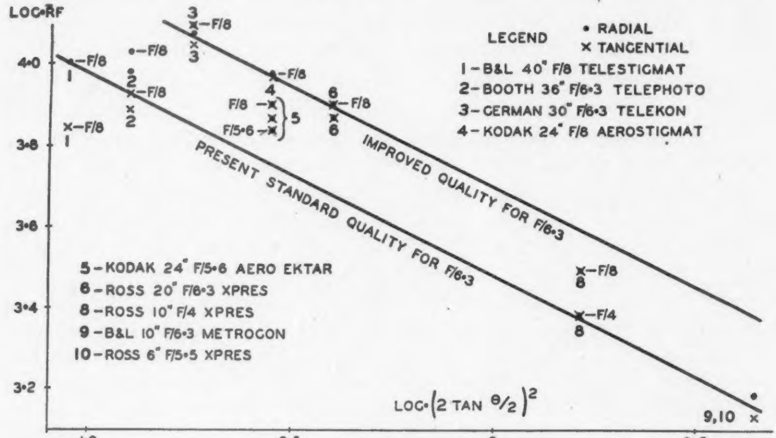

FIG. 16. *Graph for comparing the relative performance of lenses for aerial photography over a 9 by 9 in. area, on the basis of the Cobb target.*

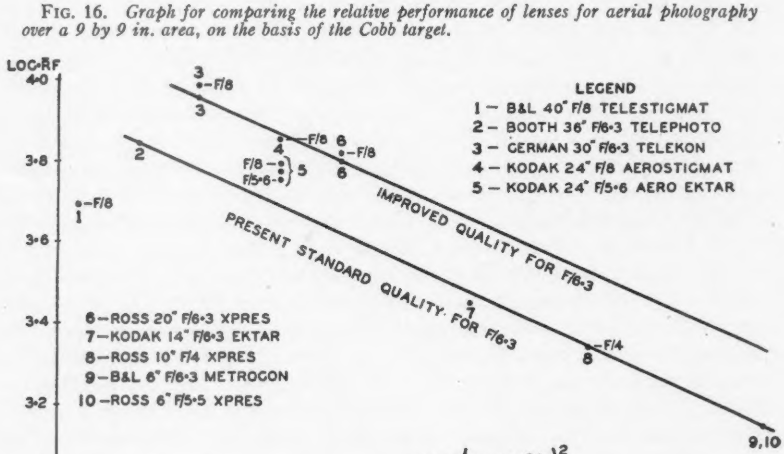

FIG. 17. *Graph for comparing the relative performance of lenses for aerial photography over a 9 by 9 in. area, on the basis of the annulus target.*

veniently made. Fig. 17 has a greater usefulness than Fig. 16. The laboratory performance shown is more nearly that which will be found in actual use. Further, the performance is stated unambiguously by a single point.

Figs. 16 and 17 are also of interest for exhibiting the different relative assessments made of the same lenses by the use of two types of targets.

The Desirable Trend in Experiment and Design

The failure of lens–film combinations used in aerial photography to attain the maximum resolving power of a photographic material is clearly due to the fact that the energy distribution in the image formed by the one is not well suited for combination with the sensitivity characteristics of the other. The blame for this condition could be placed on either the emulsion maker or the lens designer. However, it seems unlikely for a number of reasons that the sensitivity characteristics of emulsions currently used in aerial photography can be radically modified in the near future to suit the intensity distribution within the image formed by available lenses. The main reason for this conclusion is that a feasible change leading to an improvement in the resolving power of the present lens–film combination would probably reduce in other ways the usefulness of the emulsion for aerial photography. Accordingly, improved performance of aerial photographic objectives seems to depend on the full realization by designers that it is not sufficient for all purposes to design lenses having the same energy distribution in the image as that required for visual uses. Lenses for aerial photography must favour the attainment of maximum resolution of Aero Super XX, its equivalent, or the currently accepted emulsion when used in combination with it. The requirement for a lens that can be combined with the sensitivity of the average eye for attaining the maximum resolution of the latter has been much more successfully met. There seems to be no reason why a correspondingly successful result cannot be attained when the sensitivity of any other receptor is concerned. It is hardly necessary to mention that, if for a particular use a specific filter has become standard, its effect on the sensitivity characteristics of the receptor must be taken into account. In aerial photography this means currently that due consideration must be given to the modification effected on the emulsion sensitivity by Wratten No. 12 or its equivalent.

To assist the designer in developing new and more satisfactory criteria for the design of aerial photographic lenses it is essential that attention be devoted to all aspects of the image-forming properties of optical systems. The ideal solution to the problem would be the ability to predict theoretically the physical energy distribution within the image space in the region of the focal plane of a lens. With this information, a knowledge of the sensitivity characteristics of the emulsion, the quality of illumination, and the maximum and minimum exposures to be encountered in practice, it would be possible to calculate the performance of any lens–film combination. Such an accomplishment at every part of the field may be a difficult one and is not likely to be attained rapidly.

Pending a complete solution of the problem it is certain that useful progress will result from efforts to correlate empirically photographic performance, as defined by the requirements of photography, with various criteria on the desirable limitation of the residual aberrations. A similar procedure in the case of lenses intended for visual use has led to excellent results. When it is realized that the present aerial photographic emulsions in major use resolve with a low contrast test target only 22 lines per millimetre and, with a high contrast test target, only 50 lines per millimetre, the standard stated does not seem in the long run impossible of accomplishment.

Jones and Wolfe (3) recently proposed an ingenious method for studying by photographic methods the distribution of energy in the image space. The distribution patterns presented in their paper suggest possibilities of providing assistance to designers in their problem of defining desirable corrections. It should be realized however that these studies are inadequate if made only through radial and tangential sections of the image space. The distribution of energy in every direction is of equal importance for the revelation of maximum information from a photograph. Accordingly, energy distribution should be obtained by the same technique in a number of other cross-sections. Studies in six directions would probably be sufficient and, if carried out with several monochromatic sources, information useful to the designer would almost inevitably be uncovered.

When considering, from the point of view of aerial photography, the energy patterns produced by the Jones–Wolfe method it is of some importance to keep in mind that the useful exposure range in such work does not generally exceed a log exposure of 0.75, and that the desirable minimum density on the negative is 0.5 to 0.6. Consequently, the wings of the image of a point source that require very considerable overexposure to record on the emulsion of use have relatively little practical effect on the over-all resolving power of a lens–film combination.

Acknowledgments

The author acknowledges his debt to Mr. K. M. Baird of this laboratory for the experimental results used in this discussion, and to him and Mr. P. D. Carman, also of this laboratory, for many invaluable discussions.

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HIGH SPEED CAMERA[1]

By K. M. Baird[2]

Abstract

A new type, ultra high speed, motion picture camera is being developed in the Optics Section of the National Research Laboratories for the purpose of assisting research in the recently important fields of high speed aerodynamics, ballistics, and other high rate phenomena. It is expected that the camera will enable pictures having good resolving power to be taken at rates as high as 200,000 per sec. This report describes the principle of the camera and also describes a small experimental model that has been made. Pictures taken at the rate of 64,000 per sec. with the model are included.

Introduction

Generally speaking, the difficulties in high speed motion picture photography have to do with film movement, shutter mechanism, and illumination.

In taking pictures at very high speeds it is not practical to have the film travel intermittently as is customary in ordinary motion picture cameras. Moreover, at extremely high speeds, film cannot be supplied from a magazine because it would tear and too much would be used while accelerating to the proper speed. Consequently the film is usually wound in a single layer on a drum. In order to take pictures, the drum is first accelerated to the proper speed, then all the pictures are taken in a single revolution.

Because of the continuous motion, relative movement of the image and the film during each exposure will cause blurred pictures, unless a moving optical system is used or unless each exposure is very short compared to the time required for the film to travel the diameter of one frame.

The moving optical system generally consists of a rotating prism or mirror, or lenses mounted on a rotating disc. The alternative of having very short individual exposures is usually accomplished by having stroboscopic illumination such as that provided by electric sparks or an electrical condenser discharging in a gas-filled tube. These sources give intense flashes of very short duration. The above-mentioned devices are used without any other shutter, except that used to ensure that the film is exposed for only one revolution of the drum (1, pp. 762–768).

However, by any of these methods it is difficult to obtain good pictures at rates in excess of 50,000 per sec. and at these speeds large, cumbersome apparatus is required and illumination is generally poor. The camera described below embodies new principles that will make possible picture rates as high as 200,000 per sec., the pictures having good resolving power and good illumination.

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Contribution from the Division of Physics and Electrical Engineering, National Research Laboratories, Ottawa, Canada. Issued as N.R.C. No. 1397.

[2] *Physicist.*

Principle of the New Camera

The principle of the new camera will be clear from Fig. 1.

A set of lenses, L, mounted in a line forms a corresponding set of images at A. These images are projected by a lens system, M, to the rotating reflecting prisms, P, and thence to the film at F. There is a separate prism corresponding to each lens, L. The film is wound in a single layer on a drum. A rotating sector, S, having equidistant radial slots, acts as a shutter. The

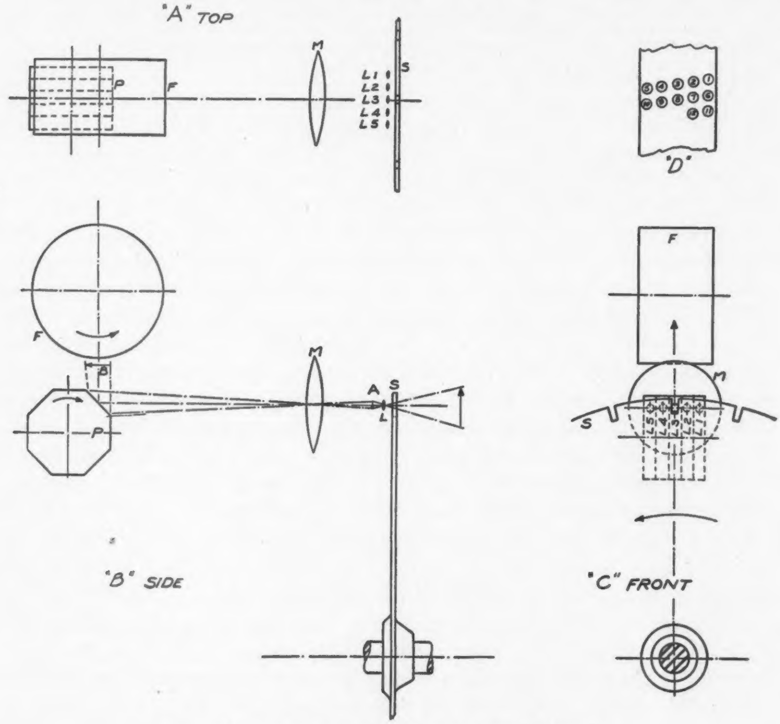

FIG. 1. *Diagrammatic views of ultra high speed camera.*

spacing of the slots is such that after a given slot passes the last lens in the row, the next slot comes opposite the first lens. Consequently, the lenses are repeatedly exposed in the order, 1, 2, 3, 4, 5. Because the film moves in the direction indicated in the figure, the resulting pictures are disposed as shown

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FIG. 2. *Photographs of test chart showing relative performances of a standard camera and the optical system to be used in ultra high speed camera.*

A—Taken with ultra high speed camera optical system.

B—Taken with a 25 mm. Kodak Anastigmat f/1.9 stopped down to f/8.

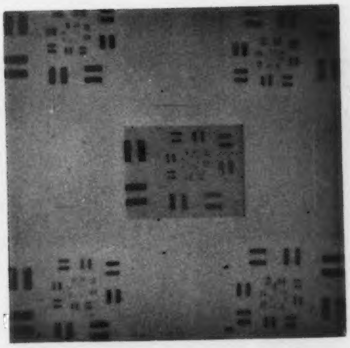

FIG. A

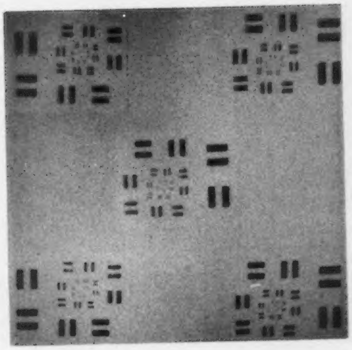

FIG B

FIG 2

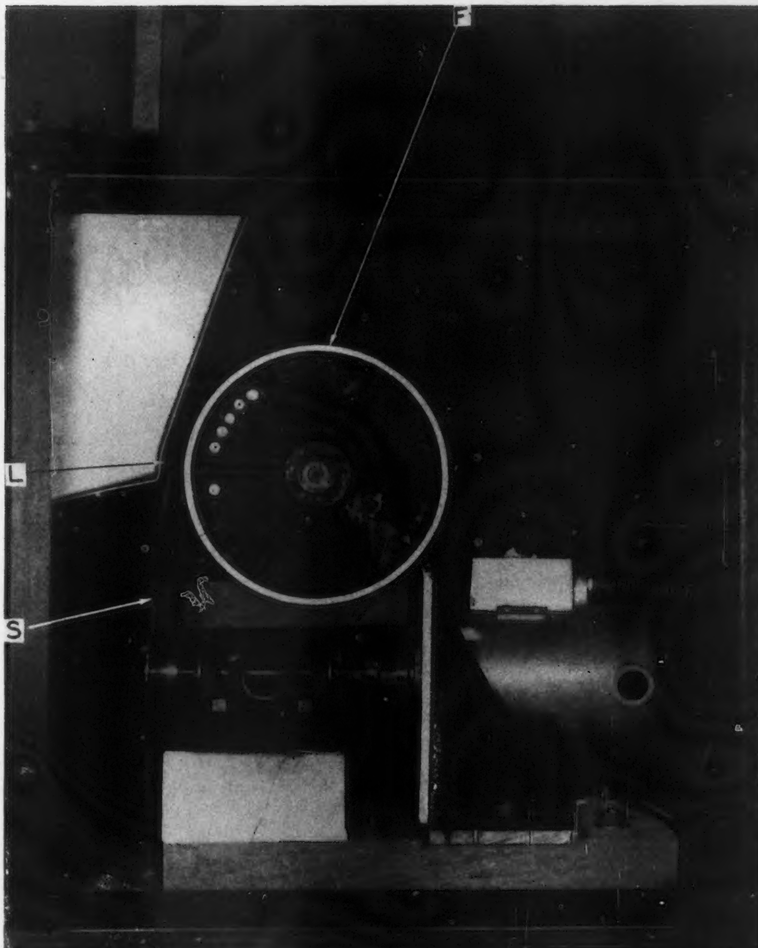

FIG. 3. *Experimental model high speed camera. L—Objective lenses. S—Sector. F—Film.*

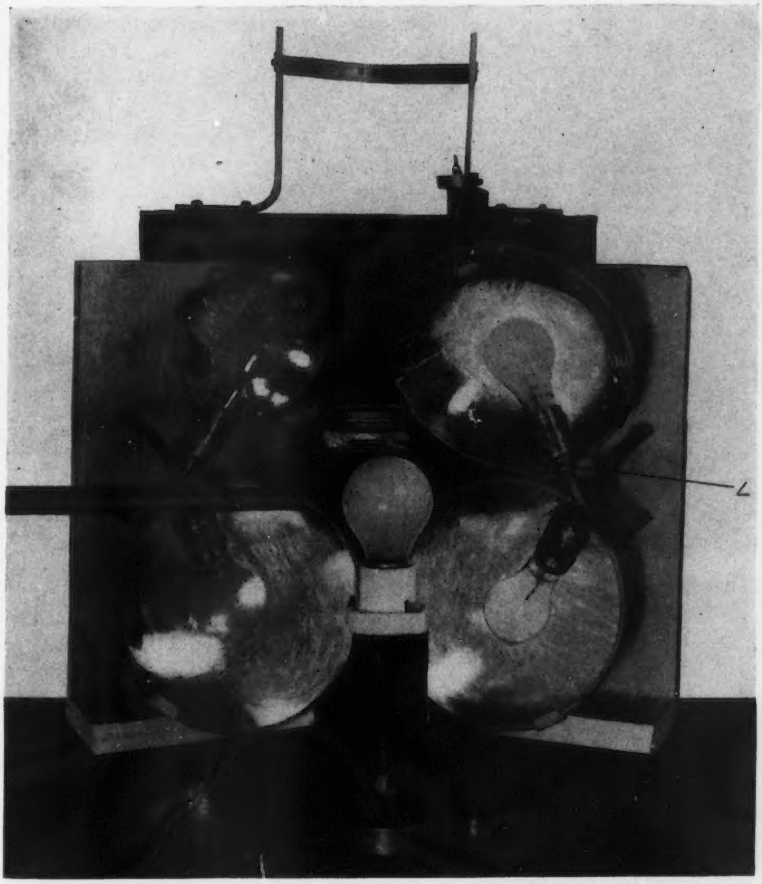

FIG. 4. *Experimental model high speed camera showing method of illuminating subject with flash bulbs. L—Objective lenses.*

in Fig. 1, *D*. The numbers indicate the order in which the pictures are taken. The rotating prisms, *P*, cause the images at *B* to move with the film, thus preventing blurring from relative motion of film and image during exposure.

It is clear that the individual picture size at *A* determines the spacing of lenses, *L*, and therefore determines the sector speed required for a given rate of taking pictures. In order to obtain a large picture size at a high rate, without unreasonably high sector speeds, the images at *A* are enlarged by the lens system, *M*, as they are projected on to the film. This enlarging is profitable because of the lens–film resolving power relations.

It is seen that by the mechanism described, the camera takes several pictures while the film moves the diameter of only one. At least three major advantages result from this. With a given film speed, pictures can be taken at several times the rate possible using the conventional method of taking only a single row of pictures; compensation for film movement is not as critical as in former types because the film moves only a fraction of the speed for a given picture rate; finally, several times as many pictures can be taken in one loading of the drum.

In order to project the pictures with a standard motion picture projector, they will be printed on to regular 16 mm. film. A machine will be made for this purpose. Pictures taken at 200,000 per sec. and projected at the standard speed of 16 per sec. will apparently retard motions by a factor of 12,500.

The design in Fig. 1 is considerably simplified for clarity. For example, in the camera being constructed there will be 10 lenses, *L*, instead of only five as shown. Also the film will be held on the inside of a drum, the images being projected via mirrors to the inside. If it were on the outside as shown, there would be danger of the film breaking under centrifugal force.

The sector, prisms, and drum will be geared together and driven by high speed electric motors. For taking pictures at a rate of 200,000 per sec. with a frame height of 0.3 in. the following rotational speeds will be used:

Sector:	30,000 r.p.m.
Prisms:	60,000 r.p.m.
Drum:	9,000 r.p.m.

For special work requiring extremely high picture rates, e.g., studying detonations or electrical discharges, the set of 10 lenses at *L* will be replaced by 40 lenses having apertures one-quarter as large, arranged in two rows of 20, one above the other. The maximum picture rate possible will then be 800,000 per sec. using the rotational speeds listed above. Both the resolving power and field of view will be considerably reduced.

Fig. 2, *A*, shows a photograph of a resolving power chart taken with the optical system to be used in the new camera. For purposes of comparison, Fig. 2, *B*, shows the same chart photographed with a Cine Kodak 16 mm. motion picture camera fitted with a 25 mm. Anastigmat $f/1.9$ lens stopped down to $f/8$. Both pictures were taken on Kodak Super XX film. It is seen that the two have comparable resolving power.

Illumination

Not much difficulty is anticipated in providing sufficient illumination for even the highest speeds. Whereas very intense illumination is required, it need have only short duration because any event requiring such a high picture rate does not last long. (The camera uses its total supply of film in 1/200 sec.) The requisite of a short, very intense flash is provided satisfactorily by electric discharge tubes, or by flash bulbs. It is estimated that discharge tubes of the type used in night aerial photography would give sufficient illumination over an area 10 ft. in diameter to take 150 pictures at the rate of 200,000 per sec. The pictures of an electric light bulb in Fig. 6 were taken at an aperture of $f/30$ with an exposure of 1/200,000 sec. The illumination was provided by

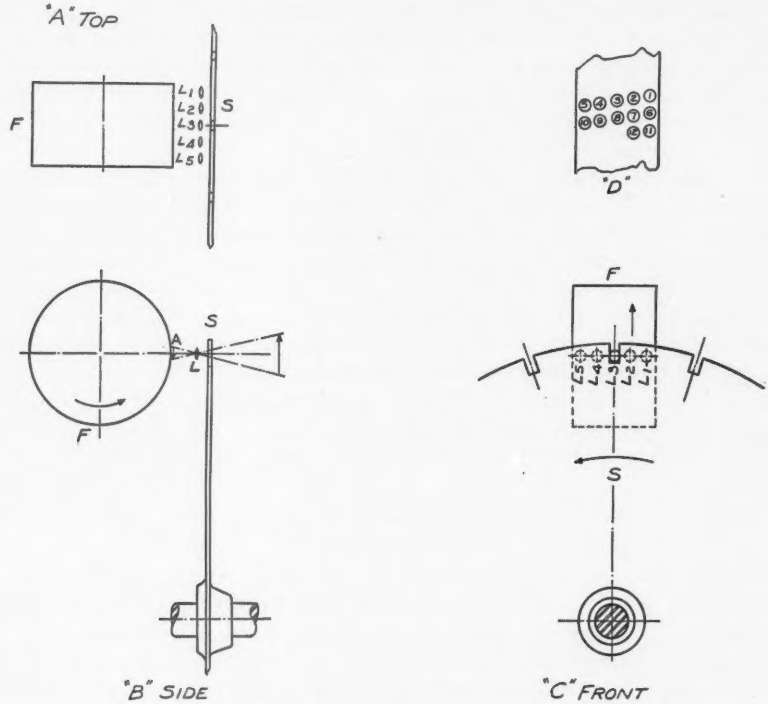

FIG. 5. *Diagrammatic views of experimental model high speed camera.*

FIG. 6. *High speed pictures of a 0·22 calibre bullet being fired through a light bulb, taken at the rate of 64,000 per sec. The order of pictures is from right to left and top to bottom. The bullet is seen to leave the muzzle of the rifle and enter the bulb in the fifth row of pictures, emerging in the seventh row.*

FIG. 7. *The bulb shown in Fig. 6 is seen disintegrating 1/150 sec. later.*

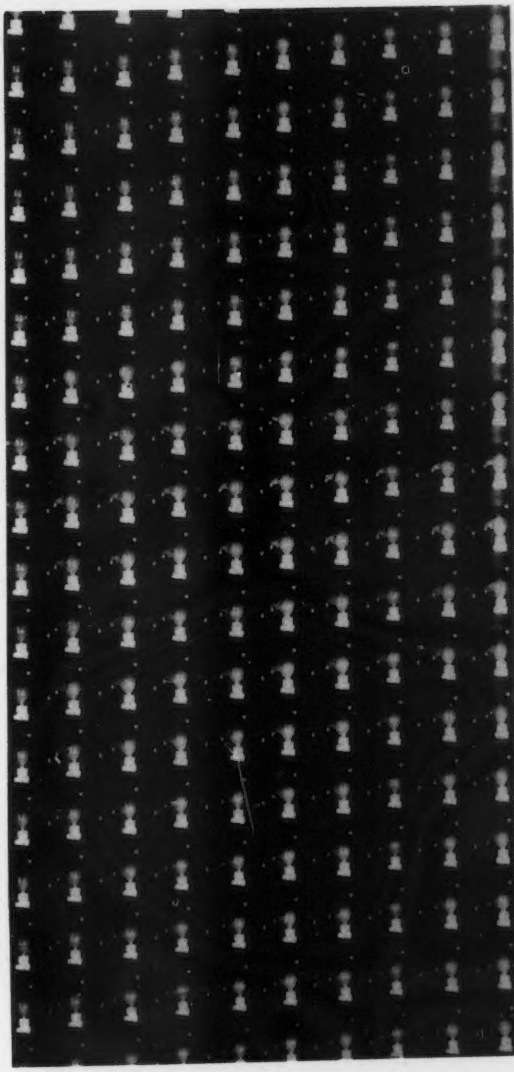

FIG. 6

PLATE V

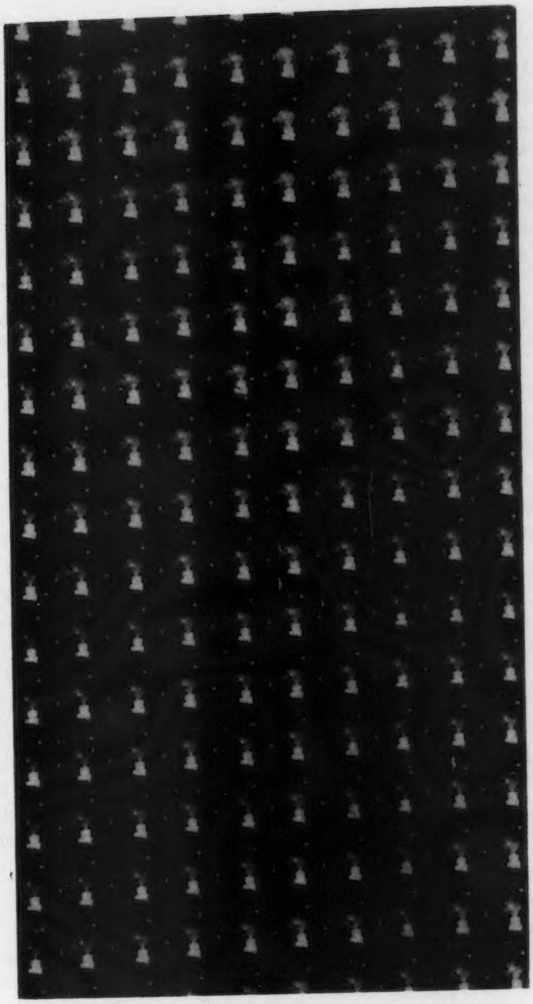

FIG. 7

flash bulbs mounted in good quality reflectors as shown in Fig. 4. It is evident that in spite of the small aperture and very short exposure the illumination was adequate.

Experimental Model

In order to test the practicability of the design of the camera the experimental model shown in Figs. 3 and 4 was built. It is shown diagrammatically in Fig. 5. It will be noted that the images formed by the lenses, *L*, are received directly onto the film, *F*. Consequently, the picture size is much smaller than in the camera described above. Also, since there is no moving optical system to compensate for film movement, the apertures of the lenses had to be made very small so that the exposure time of each picture would be short compared to the interval between successive pictures. This results in considerable loss of resolution.

However, in spite of these defects and in spite of the relatively crude construction of the camera, quite good pictures were obtained at rates over 70,000 per sec.

Some of the pictures, taken at the rate of 64,000 per sec., appear in Figs. 6 and 7. They show a 0·22 calibre rifle bullet fired into a light bulb. The order of pictures is from right to left and from top to bottom. The muzzle of the rifle is seen at the right-hand side of each picture. In the fifth row from the top of Fig. 6 the bullet is seen to leave the rifle and enter the bulb. It emerges from the bulb in the seventh row. Fig. 7 shows the bulb disintegrating 1/150 sec. later.

The arrangement used to take the pictures is shown in Fig. 4. The sector and drum were accelerated to speed, then the flash bulbs and bullet fired simultaneously. The flash bulbs gave sufficient illumination for about 20 milliseconds, in which time over 1200 pictures were taken.

Pictures taken with the experimental camera have been successfully projected as motion pictures.

As this is an interim report, the principle of the new camera has been described very briefly and the mechanical design omitted entirely. A more detailed description of the camera and its characteristics will be given in a later publication.

Acknowledgment

The author wishes to express his gratitude to Dr. L. E. Howlett of this laboratory for suggesting the problem of an ultra high speed camera and for his encouragement and assistance in developing it.

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SECONDARY RADIATION FROM X-RAY FILTERS

II. THE DESIGN OF EFFICIENT MULTIMETAL FILTERS[1]

By N. M. Morrow[2]

Abstract

An experimental procedure for the design of efficient multimetal X-ray filters is presented. The applicability of such procedure is demonstrated by designing a filter for use with radiation from a Coolidge type X-ray tube operated at 100 kvp., for which it is found that 0.043 gm. per cm.^{2} of copper backed by 0.08 gm. per cm.^{2} of aluminum is highly effective.

Introduction

When X-radiations are used in deep therapy work, it is desired to deliver a large radiation dosage to the diseased tissue, without undue damage to the surface and healthy tissues. One method of reducing the damage to surface tissue is to remove from the X-ray beam, by filtration, a large part of the softer radiation.

In terms of physical observables, the features desired in an X-ray filter are:

(*a*) That the intensity, I, of the primary X-ray beam transmitted by the filter should be as large as possible relative to the intensity, I_0, incident on the filter.

(*b*) That the increase in depth dose* $(D - D_0)$ of the X-radiation, due to the selective absorption of the filter, should be as large as possible. The depth dose with no filter is represented by D_0, and that with the filter, by D.

(*c*) In addition to these principal factors, it is highly desirable that the forwards transmitted secondary radiations from the filter be minimized. This is particularly true of the softer and therefore more dangerous fluorescence radiations originating in the filter which may cause severe surface burns.

Increased depth dose may be obtained by increasing the thickness of a given filter, or by using composite filters. Such filters have been studied by many writers (2, 3, 5, 6).

The present investigation was undertaken in an effort to develop an impersonal procedure for construction of a filter in which the above features would be incorporated as effectively as possible.

Examination of the physical laws governing X-ray absorption reveals that features (*a*) and (*b*) cannot be obtained in greatest measure simultaneously. Thus if the depth dose increase were to be made very large, a thick filter would

[1] *Manuscript received March 6, 1946.*

Contribution from the Department of Physics, McMaster University, Hamilton, Ontario. This problem was suggested by Dr. G. A. Wrenshall of the Banting Institute, University of Toronto, and was done under his supervision. It represents a sequel to Part I by Wrenshall and Nichols (7).

[2] *Graduate Research Student, holder of a Bursary under the National Research Council of Canada.*

* *Depth dose may be defined as the ratio of the intensity of X-rays at a depth of 10 cm. below the surface, to that at the surface (4), i.e.* $D = I_{10}/I$.

be required which would cause I/I_0 to be very small. Conversely, if a thin filter were used so that I/I_0 was large, the increase in depth dose would be very small. However, a useful compromise between these two limiting conditions can be made by means of the following definition. If the efficiency E of a filter is defined as

$$E = (D - D_0)I/I_0$$

it will have a maximum value for intermediate values of $(D - D_0)$ and I/I_0, and will represent the best coexistent values of these variables, if they are considered to be of equal importance.

Using this concept of efficiency it becomes possible to construct a filter that will have a maximum efficiency for a given beam of heterogeneous X-rays. The steps required in the actual assembly of such a filter are presented in this paper for 100 kvp. X-rays from a tungsten target Coolidge tube, as an example of the applicability of the method. In addition, steps are described by which the intensity of soft forwards transmitted secondary X-rays from the filter can be minimized.

Experimental

The following procedure is used in this paper. (*a*) The values of $(D - D_0)$ and I/I_0 are measured for various thicknesses of each of the single metals considered for use in the filters. From this the $E_{max.}$ value is noted for each metal. (*b*) The metal having highest $E_{max.}$ is selected and backed by various thicknesses of metals of lower atomic number. This backing should cause a notable increase in efficiency by heavily absorbing any soft primary radiation transmitted by the original filter owing to absorption discontinuities. In addition it should be highly effective in the absorption of the dangerous components of the secondary radiation emitted by the filter. In this way each of the desirable features listed above for X-ray filtration can be incorporated efficiently and impersonally into a filter designed for use with a specific type of radiation.

The general arrangement of the apparatus employed in this study has, with the exception of certain modifications outlined below, been described in an earlier paper (7), and illustrated in Fig. 2 of that paper.

The following types of measurements were made and are reported in this paper:

Type I. The depth dose was measured for each of the filters. The experimental arrangement is shown in Fig. 1. The depth dose was measured in a water phantom* N, which consisted of a rolled iron container 10 cm. in length and 12.5 cm. in diameter. The ends consisted of thin celluloid sheets, and the phantom was completely filled with water. The ionization chamber was lowered so that the primary beam passed through the aluminum window, H, in the ionization chamber. A flat annular ring of lead, M, 2 mm. thick, was

** Cipollaro and Mutscheller (1) have shown that absorption data obtained with water is directly applicable to live skin tissue.*

placed over the chamber window. This ring contained a hole sufficiently large for the primary beam to pass through unobstructed, but small enough to prevent most of the secondary radiation from the filter, *F*, from entering the chamber. X-ray intensities were measured with and without the phantom in place, and the depth dose $D = I_{10}/I$ was calculated.

Type II. The intensity of the transmitted primary beam was measured for each of the filters.

Type III. The intensity of the secondary radiation was measured for each filter.

Type IV. The space rate of absorption in aluminum of the secondary radiation from the filter was measured. The arrangement used is shown in Fig. 2.

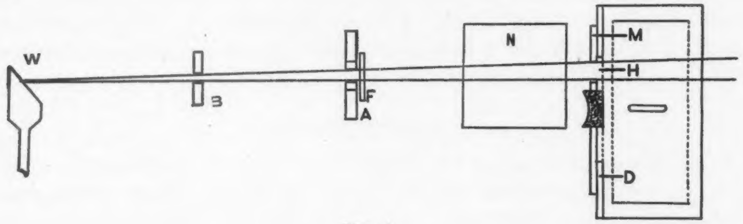

FIG. 1.

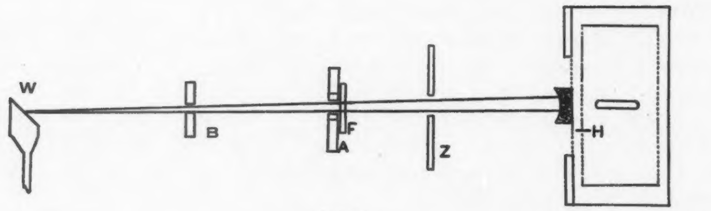

FIG. 2.

FIGS. 1 and 2. *Geometric arrangements used for the X-ray source W, apertures A and B, filter F, water phantom N, lead shield M, aluminum absorbers Z, and ionization chamber.*

The procedures followed in measurements of Types II, III, and IV have been described previously (7). All intensities reported were measured relative to that of the unfiltered primary beam, which was held at a constant value throughout the observations.

Results

The efficiency of the single metal filters of lead, tin, copper, and aluminum is shown for the X-ray beam employed, in Fig. 3, as a function of the mass per unit area of filter. A definite maximum value of E exists for each metal. In the case of aluminum the maximum ($E = 0.0105$) occurred at a mass per unit area of 0.50 gm. per cm.^{2} Lead and copper seem to offer the best possi-

bilities since they give the highest efficiency E. However, the thickness of lead required to give high efficiency is so small that it would be almost impossible to produce large areas of uniform thickness. Copper, on the other

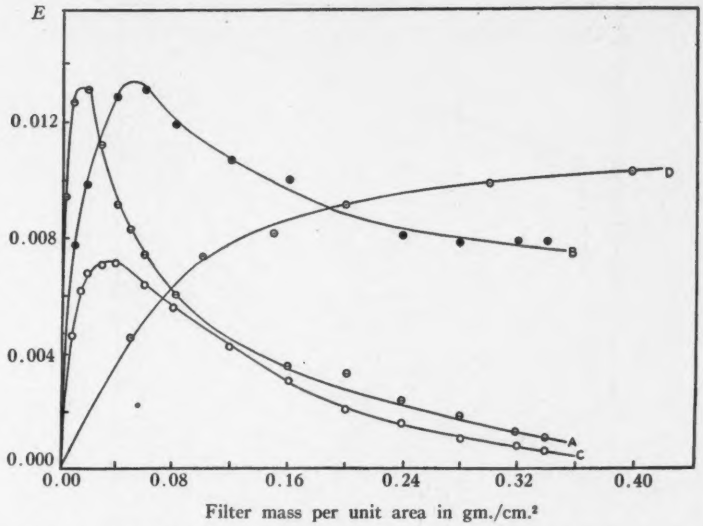

FIG. 3. *Efficiency, E, versus filter mass per unit area for 100 kpv. X-rays, A—lead. B—copper. C—tin. D—aluminum.*

hand, gives high efficiency at thicknesses that may readily be produced commercially (0.0432 gm. per cm.²). Comparing these efficiency curves with the curves of Wrenshall and Nichols (7), it may be seen that in each case the mass per unit area which gives maximum efficiency is approximately the same as that which gives a maximum of secondary radiation from the filter. Therefore to use such filters to advantage, this secondary radiation should be largely removed. This shows the need for the use of composite filters.

Using measurements of Type IV the change in quality of the secondary radiation from the copper was studied as a function of the mass per unit area of the copper. The secondary radiation was absorbed in various thicknesses of aluminum and $-\text{Log } I_s/I_{so}$ was plotted as a function of the mass per unit area of the aluminum. I_s is the intensity of the secondary radiation reaching the chamber after passing through a given aluminum absorber, while I_{so} represents the intensity with no absorber. If the radiation under examination is homogeneous, the resulting graph should be a straight line. The results are shown in Fig. 4. Each graph consists of two parts, each of which is approximately a straight line, indicating that the secondary radiation consists of two components of different wave-lengths, each almost homogeneous. The steep part corresponds to the absorption in aluminum of the soft copper fluorescence

radiation, while the second part with a smaller slope corresponds to the absorption of the harder scattered radiation. The percentage of copper fluorescence radiation in the secondary radiation was estimated as follows,

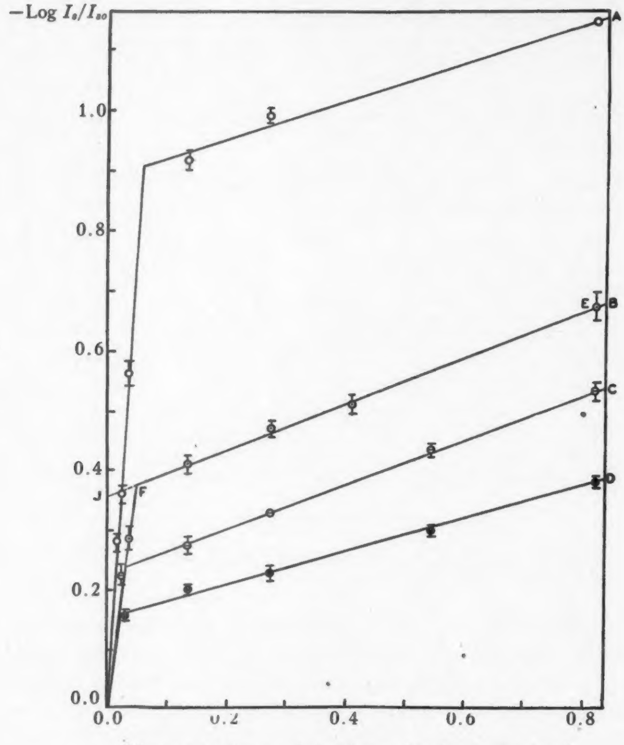

FIG. 4. — *Log* I_s/I_{so} *versus mass per unit area of aluminum absorber for 100 kvp. X-rays. A—0.0083 gm. per cm.² of copper. B—0.0903 gm. per cm.² of copper. C—0.1537 gm. per cm.² of copper. D—0.3588 gm. per cm.² of copper.*

referring to curve B of the graph. The part EF of the curve was produced to cut the $-\text{Log } I_s/I_{so}$ axis at J. If it is assumed that J represents the value of $-\text{Log } I_s/I_{so}$ when I_s is the intensity of scattered radiation alone, then

$$- \text{Log } \frac{\text{total intensity of secondary radiation}}{\text{intensity of scattered radiation}} = J.$$

The percentage of copper fluorescence radiation was then plotted as a function of the mass per unit area of the copper filter in Fig. 5. This curve shows that for very thin copper filters the secondary radiation is almost entirely fluorescence radiation, while, as the thickness of copper increases,

the percentage of fluorescence radiation drops off rapidly at first and then more slowly. Therefore, the thicker the copper filter, the less dangerous is the secondary radiation. The actual percentage values are not accurate

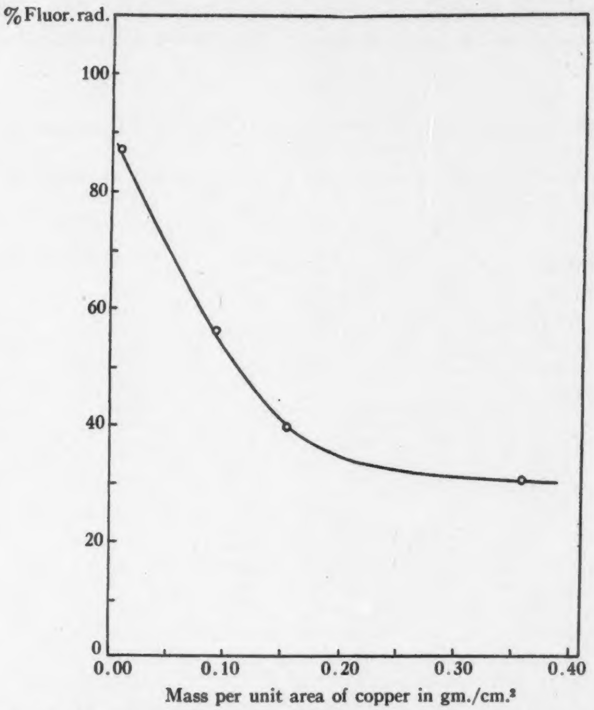

FIG. 5. *Percentage of fluorescence radiation in the secondary radiation from copper versus mass per unit area of copper for 100 kvp. X-rays.*

owing to the difficulty in comparing intensities of radiations of two different wave-lengths by the ionization chamber method. This difficulty, however, will not cause any material change in the shape of the curve of Fig. 5, nor will it have an effect if a filter can be constructed for which one of the components can be reduced to a negligible intensity compared with the other.

Using measurements of Types I and II the ratio I/I_0 of the primary beam, the relative depth dose, and the filter efficiency were calculated for the following systems of composite filters:

(i) 0.0432 gm. per cm.² of copper backed by various thicknesses of aluminum. This mass per unit area of copper was found to give a maximum efficiency for single metal filters.

(ii) 0.0231 gm. per cm.² of copper backed by various thicknesses of aluminum.

(iii) 0.2731 gm. per cm.^{2} of copper backed by various thicknesses of aluminum.

The results of these measurements are presented in Table I. The backing of copper by aluminum in each case causes an increase in depth dose. This increase in depth dose is due to three effects: (*a*) ordinary filtering action of

TABLE I

DEPTH DOSE (I_{10}/I), I/I_0 OF THE PRIMARY BEAM, AND EFFICIENCY (E) FOR VARIOUS COMPOSITE FILTERS

Filter	I/I_0	I_{10}/I	E
Cu—0.0231 gm./cm.^{2}	0.56	0.043	0.0086
+ Al—0.0352 gm./cm.^{2}	.50	.050	.0112
+ Al—0.0799 gm./cm.^{2}	.44	.052	.0109
+ Al—0.1325 gm./cm.^{2}	.40	.056	.0114
+ Al—0.2651 gm./cm.^{2}	.342	.061	.0115
+ Al—0.454 gm./cm.^{2}	.260	.069	.0107
Cu—0.0432 gm./cm.^{2}	.376	.055	.0105
+ Al—0.0352 gm./cm.^{2}	.395	.058	.0110
+ Al—0.0799 gm./cm.^{2}	.344	.060	.0110
+ Al—0.1325 gm./cm.^{2}	.313	.065	.0118
+ Al—0.2651 gm./cm.^{2}	.244	.071	.0106
+ Al—0.454 gm./cm.^{2}	.176	.080	.0092
Cu—0.2731 gm./cm.^{2}	.092	.109	.0075
+ Al—0.0352 gm./cm.^{2}	.093	.111	.0076
+ Al—0.0799 gm./cm.^{2}	.090	.111	.0075
+ Al—0.1325 gm./cm.^{2}	.085	.122	.0080
+ Al—0.2651 gm./cm.^{2}	.082	.114	.0070
+ Al—0.454 gm./cm.^{2}	.068	.114	.0058

the aluminum as is observed in single metal filters, (*b*) heavy absorption in aluminum of the soft radiation (1.38 to 2.5 Å), which readily passes through the copper owing to its absorption discontinuity, (*c*) heavy absorption in aluminum of the soft secondary copper fluorescence radiation. The thinner the original copper filter, the greater is the rise in depth dose when aluminum is added. The efficiency of the filter also rises with the first small addition of aluminum backing, but then remains constant within experimental error until the mass per unit area of aluminum is approximately 0.45 gm. per cm.^{2} and then again begins to drop.

Using measurements of Type III the intensity of the forwards transmitted secondary radiation from each of the above composite filter systems was examined. The reference filter was in all cases the same, so intensities may be directly compared. The results are shown in Fig. 6.

As the thickness of aluminum backing the copper filter is increased, the intensity of secondary radiation drops sharply, reaches a minimum, and then increases again. Since the copper fluorescence radiation is very soft, it is heavily absorbed in a very thin piece of aluminum and it is this removal of

the copper fluorescence radiation that causes the original sharp drop in intensity. However, as the thickness of aluminum is increased, there will be more X-rays scattered from the aluminum, and this will tend to increase

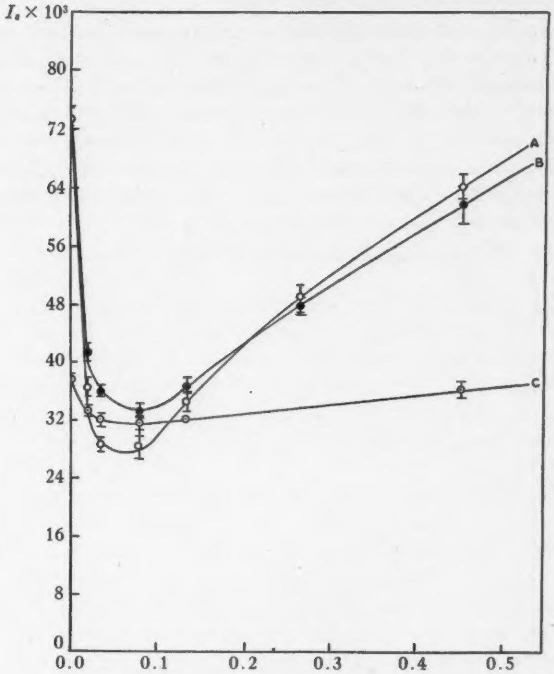

FIG. 6. *Intensity of secondary radiation, I_s, from a copper filter versus mass per unit area of aluminum backing the copper, for 100 kvp. X-rays. A—0.0231 gm. per cm.² of copper. B—0.0432 gm. per cm.² of copper. C—0.1732 gm. per cm.² of copper.*

the intensity of secondary radiation from the filter (see reference (7)). A point is reached where the increase in scattered radiation overcomes the decrease in fluorescence radiation and the intensity of secondary radiation begins to rise. It is advisable to use an aluminum backing of such thickness that the intensity of secondary radiation is a minimum. The use of a thinner backing would let through an important fraction of the dangerous copper fluorescence radiation that is so undesirable. The use of a thicker aluminum backing would cause an increase in the intensity of the secondary radiation from the filter. This increase is due to scattered radiation, and, since it is of about the same hardness as the primary beam, it is no more dangerous with regard to burning effect. However, since radiation is scattered in all directions, increased scattered radiation represents energy lost from the

primary beam, and therefore should be avoided. For the three cases studied, 0.08 gm. per cm.^{2} of aluminum is sufficient to cause a minimum of secondary radiation. This is also found to be the thickness that gives highest efficiency, although this effect on efficiency is quite small.

From measurements of Type IV, the secondary radiation from a copper filter of 0.0432 gm. per cm.^{2} backed by 0.08 gm. per cm.^{2} of aluminum was examined to determine how much of the copper fluorescence had been removed by the aluminum. The results are shown in Fig. 7. No original steep part is present as was observed for pure copper filters. This means that the 0.08 gm. per cm.^{2} of aluminum effectively eliminated the copper fluorescence radiation from the filter. Theoretically all but 2% of the copper fluorescence radiation was eliminated.

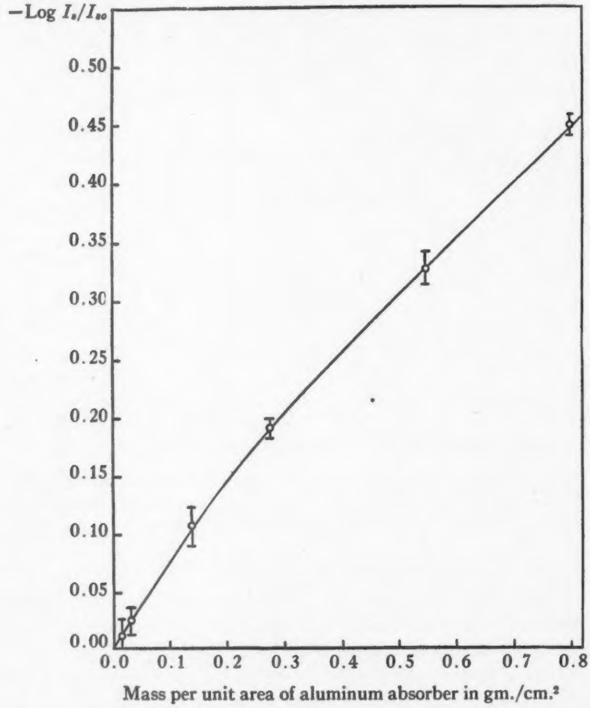

FIG. 7. — *Log I_s/I_{s0} versus mass per unit area of aluminum absorber for the secondary radiation from a copper filter of 0.0432 gm. per cm.^{2} backed by 0.0799 gm. per cm.^{2} of aluminum for 100 kvp. X-rays.*

Therefore a filter composed of 0.0432 gm. per cm.^{2} of copper backed by 0.08 gm. per cm.^{2} of aluminum is the most effective filter for use with the 100 kvp. X-ray source employed, in that it best satisfies the criteria set forth

in this paper. It gives high efficiency, with a minimum intensity of secondary radiation, and the dangerous copper fluorescence radiation is effectively eliminated.

Conclusions

A practical experimental procedure has been drawn up for designing a compound filter that provides certain desirable features of a filter; and its applicability has been demonstrated for 100 kvp. X-radiation from a Coolidge type tube. This same general procedure can be used for designing filters for use with other voltages as desired.

For each of the single metal filters considered, a filter thickness exists that gives a maximum filter efficiency, and such a filter should be used for filtration. This thickness, however, gives a maximum intensity of secondary radiation from the filter, and this should be absorbed in some other metal to increase the effectiveness of the filter.

Copper gives the most efficient and most practical filter for 100 kvp. X-rays, provided its soft fluorescence radiation is absorbed in aluminum. As the thickness of the copper filter increases, the percentage of copper fluorescence radiation in the secondary radiation decreases.

Tin filters are very inefficient for 100 kvp. radiation. The advantage to be gained by such filters is apparently limited to use with higher voltage radiations.

Lead filters are efficient, but the thickness required to give good efficiency is too small to be practical.

Aluminum filters are reasonably efficient but are not as good as copper, and the depth doses obtained are relatively small.

For use with 100 kvp. X-radiation, it is found that 0.043 gm. per $cm.^2$ of copper backed by 0.08 gm. per $cm.^2$ of aluminum gives the most efficient filter, with a low intensity of forwards transmitted secondary radiation, while the dangerous copper fluorescence radiation is effectively eliminated.

Acknowledgments

The author wishes to express his appreciation to Dr. G. A. Wrenshall, who directed the problem, and to Prof. H. F. Dawes, of McMaster University, who supervised the work at the University. Their encouragement, advice, and helpful criticisms during the course of the work were invaluable.

The writer also wishes to thank sincerely the National Research Council of Canada for a bursary, which enabled him to take part in the work.

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TERMINATION EFFECTS IN FEEDBACK AMPLIFIER CHAINS[1]

By ALEX. J. FERGUSON[2]

Abstract

The theoretical response curves of finite chains of the feedback networks described by Wheeler are studied. A generalization of a standard formula for passive filters is obtained and applied to the problem. The paper presents computed reflection coefficients for various simple terminations and computed response curves for feedback chains of three, four, and five stages, using these terminations. Many of the response curves are unsatisfactory. These occur with the simpler types of terminations such as normally occur in practice. It is shown how matching at both ends of the amplifier can greatly improve the response curves with a loss of gain. Curves are also given that show the advantage of using a constant-*k* half-section as a termination.

Introduction

In 1939, H. A. Wheeler (3) published a classic paper on wide-band amplifier design that showed: (1) that, by the use of infinite filters, an impedance whose magnitude is constant over a wide band of frequencies can theoretically be developed, and (2) that an active four terminal device, called a "bidirective transconductance", when associated with simple circuits, has image impedances identical with those of constant-*k* filter networks and produces a constant amplification over its bandwidth. The bidirective transconductance can consequently be used in conjunction with constant-*k* filters, and Wheeler uses them in this way to obtain amplifying networks of large bandwidth.

Obviously the filter-like properties of the networks allow them to be associated with each other to form a feedback chain. If a tube with a resistance between plate and grid is used to approximate the ideal bidirective transconductance, then such a chain will take the form shown in Fig. 1, and represents a very practical way of utilizing the high performance of the circuits. Some care must be exercised in the analysis of these circuits because the networks are not passive and one cannot indiscriminately apply results that have been developed for passive ones. For this reason, the analysis will be developed *ab initio*. However, there is between the passive case and the more general active case a very close analogy that is not made entirely clear in Wheeler's paper and that will appear in the formulae developed here.

The solution of the network equations can be resolved into a wave advancing from the input to the output, and another wave reflected back through the amplifier to the input. Reflection coefficients can be derived for the source and terminal impedances which allow the general equations to be reduced to a form practically identical with the passive case. The consequences of having reflected waves will generally be undesirable, so that the design will be studied

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[2] *Physicist.*

from the point of view of eliminating them. Thus the best response will be regarded as that of an amplifier with an ideal termination, i.e., a termination producing no reflections.

The method of the present investigation is to take a number of simple terminations, some of which occur in practice and others which could be readily built, to compute the frequency characteristics of their reflection coefficients and to compute the frequency responses of amplifiers using these terminations. While these results refer specifically to amplifiers with a stage gain of nine, the general features will be approximately the same for other gain values. Fig. 1 shows a video amplifier, but the theory applies equally well to an i.f. amplifier, and it was actually in this connection that the investigation was undertaken.

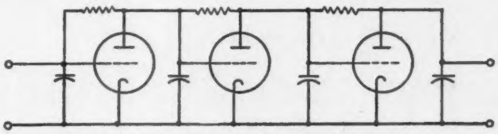

FIG. 1. *Typical feedback amplifier chain.*

Many of the amplifier responses are unsatisfactory. These curves are presented to illustrate the effects of serious mismatches. In general, a system that is approximately matched at both input and output will be considerably better than one approximately matched at one end only, and this arrangement is to be recommended as the simplest and most satisfactory. It has the disadvantage that half of the gain is lost. A slightly more elaborate termination, roughly equivalent to an additional constant-k half-section, is proposed and studied.

Deduction of Formulae

Consider a general linear network shown in Fig. 2. Let the general equations be

$$I_1 = y_{11}E_1 + y_{12}E_2 \tag{1}$$

$$I_2 = y_{21}E_1 + y_{22}E_2 \tag{2}$$

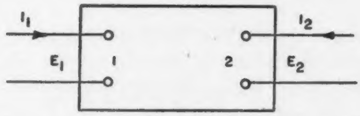

FIG. 2. *General active four-terminal network.*

It is going to be assumed throughout the work that the network is not passive, i.e.,

$$y_{12} \neq y_{21}.$$

The analysis is practically identical with that for the passive case, so that only the final results will be quoted. For an active network four parameters

are required and these will be taken to be the image admittances Y_{I1} and Y_{I2} and the two image transfer constants, θ_{12} and θ_{21}. The image transfer constants are defined by

$$e^{\theta_{12}} = \frac{E_2}{E_1} \tag{3}$$

for a generator applied to terminals 1 and an admittance Y_{I2} to terminals 2, and

$$e^{\theta_{21}} = \frac{E_1}{E_2} \tag{4}$$

for a generator applied to terminals 2, and an admittance Y_{I1} to terminals 1.

These parameters may be expressed in terms of the network constants:

$$Y_{I1}^2 = \frac{y_{11}}{y_{22}} (y_{11}y_{22} - y_{12}y_{21}) \tag{5}$$

$$Y_{I2}^2 = \frac{y_{22}}{y_{11}} (y_{11}y_{22} - y_{12}y_{21}) \tag{6}$$

$$\cosh^2 (\theta_{12} - \alpha) = \cosh^2 (\theta_{21} + \alpha) = \frac{y_{11}y_{22}}{y_{12}y_{21}}, \tag{7}$$

$$\text{where } \alpha = \log \sqrt{\frac{y_{21}y_{11}}{y_{12}y_{22}}}.$$

Equation (7) may be written in a slightly different form:

$$\tanh^2 (\theta_{12} - \alpha) = \tanh^2 (\theta_{21} + \alpha) = 1 - \frac{y_{12}y_{21}}{y_{11}y_{22}}$$

$$= \frac{Y_{oc}}{Y_{sc}}, \tag{8}$$

where Y_{oc} and Y_{sc} are respectively the open and short circuit admittances for either end.

Equations (7) and (8) have the disadvantage that a number of extraneous roots have been introduced in their construction. Thus the equation:

$$\cosh^2 \theta = \frac{y_{11}y_{22}}{y_{12}y_{21}}$$

is also satisfied by $-\theta, \theta + j\pi, -\theta + j\pi$. Assuming that θ is the appropriate root, then

$$\theta_{12} = \theta + \alpha, \quad \theta_{21} = \theta - \alpha.$$

It is of interest to examine the quantity α.

$$\alpha = \log \sqrt{\frac{y_{21}}{y_{12}}} + \log \sqrt{\frac{y_{11}}{y_{22}}}.$$

The first term on the right-hand side is not present in passive networks since then $y_{12} = y_{21}$. This term arises from the fact that the network is active and may thus be regarded as the amplification. The second term represents an increase or decrease in voltage due to a difference in the impedance level between the output and input. The image transfer constant of standard filter theory is the arithmetic mean of θ_{12} and θ_{21} and is independent of impedance transformations (see e.g. Starr (2)).

To complete the general analysis consider the network fed by a source of internal admittance Y_G and terminated by an admittance Y_T, as shown in Fig. 3. If the source has a voltage E_0 then we may show that

$$\frac{E_2}{E_0} = \frac{Y_G}{Y_G + \dfrac{Y_{I1} Y_T}{Y_{I2}}} \cdot e^{\theta_{12}} \cdot \frac{1 - r_1 r_2}{1 - e^{\theta_{12} + \theta_{21}} r_1 r_2}, \tag{9}$$

$$\text{where } r_1 = \frac{Y_G - Y_{I1}}{Y_G + Y_{I1}}$$

$$r_2 = \frac{Y_T - Y_{I2}}{Y_T + Y_{I2}}.$$

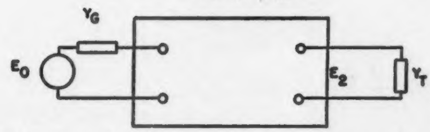

FIG. 3. *General active four-terminal network with source and load connected.*

r_1 and r_2 are the reflection coefficients associated with the source and terminal impedances. This formula is very similar to a corresponding one for passive networks (Guillemin (1, p. 281)).

If the network is composed of a number of four-terminal elements correctly joined on an image basis, then θ_{12} and θ_{21} will be the sum of the corresponding parameters for the individual elements.

This paper is concerned with cascades of vacuum tube networks such as shown in Fig. 4. The network consists of a tube of pentode type with a

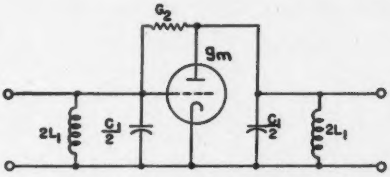

FIG. 4. *Practical intermediate frequency feedback unit.*

transconductance g_m, a pure conductance G_2 providing the feedback and pure reactive elements in the plate and grid circuits, consisting of a capacity $C_1/2$ and an inductance $2L_1$. Elements immaterial to the analysis, such as screen grids, blocking condensers, etc., have not been shown.

The admittance in the plate and grid circuits is

$$j\omega \frac{C_1}{2} + \frac{1}{2j\omega L_1} = j\omega_0 \frac{C_1}{2} x,$$

$$\text{where } x = \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}$$

$$\text{and } \omega_0 = \frac{1}{\sqrt{L_1 C_1}}.$$

All the curves will be presented in terms of the function x. In the case of an i.f. amplifier, $x = 0$ corresponds to the mid-band point and the response, in terms of x, is symmetrical about zero. The actual frequency, $\frac{\omega}{2\pi}$, is an approximately linear function of x over the bandwidth, so that the response in terms of frequency will have almost the same shape as when plotted against x.

The equations for the network of Fig. 4 are

$$I_1 = \left(j\omega_0 \frac{C_1}{2} x + G_2\right) E_1 - G_2 E_2 \tag{10}$$

$$I_2 = (g_m - G_2) E_1 + \left(j\omega_0 \frac{C_1}{2} x + G_2\right) E_2 . \tag{11}$$

These equations define the network parameters. It will be observed that $y_{11} = y_{22}$, so that $Y_{I1} = Y_{I2}$, and the networks are correctly matched when joined in cascade. The common value of the image admittances is

$$Y_I^2 = \left(j\omega_o \frac{C_1}{2} x + G_2\right)^2 + G_2 (g_m - G_2) . \tag{12}$$

This has the same form as for a constant-k network with dissipation.

For the image transfer constants we have

$$\cosh^2 \theta = \frac{-\left(j\omega_0 \frac{C_1}{2} x + G_2\right)^2}{G_2(g_m - G_2)}$$

or

$$\sinh^2\left(\theta - \frac{j\pi}{2}\right) = \frac{\left(j\omega_0 \frac{C_1}{2} x + G_2\right)^2}{G_2(g_m - G_2)}$$

and

$$\alpha = \log \sqrt{\frac{g_m - G_2}{-G_2}} = \frac{j\pi}{2} + \log \sqrt{\frac{g_m - G_2}{G_2}} .$$

It will be convenient to put

$$x_0 = \frac{2\sqrt{G_2 (g_m - G_2)}}{\omega_0 C_1} .$$

Then

$$\theta_{12} = \sinh^{-1}\left(j\frac{x}{x_0} + \frac{1}{\sqrt{\frac{g_m}{G_2} - 1}}\right) + j\pi + \log\sqrt{\frac{g_m}{G_2} - 1} \tag{13}$$

$$\theta_{21} = \sinh^{-1}\left(j\frac{x}{x_0} + \frac{1}{\sqrt{\frac{g_m}{G_2} - 1}}\right) - \log\sqrt{\frac{g_m}{G_2} - 1} . \tag{14}$$

These formulae, used with Kennelly's *Chart Atlas of Complex Hyperbolic and Circular Functions*, are the most practical for calculating numerically the image transfer constants.

The mid-band stage gain of the amplifier is

$$[e^{\theta_{12}}]_{x=0} = \exp\left[\sinh^{-1}\left(\frac{g_m}{G_2} - 1\right)^{-1/2} + \frac{1}{2}\log\left(\frac{g_m}{G_2} - 1\right)\right] = \sqrt{\frac{g_m}{G_2}} - 1\,.$$

The results of this report are for the specific case where $\frac{g_m}{G_2} = 100$, i.e. a stage gain of nine. The general features for other cases will be very much the same, and the conclusions drawn from the following curves will always apply.

If the amplifier has N stages, each with image transfer constants θ_{12} and θ_{21}, then Equation (9) becomes

$$E_2 = \frac{E_0 Y_G}{Y_G + Y_T} \cdot e^{N\theta_{12}} \cdot \frac{1 - r_1 r_2}{1 - e^{N(\theta_{12} + \theta_{21})} r_1 r_2}\,. \tag{15}$$

Certain modifications and special cases of this formula are of interest.

1. Source of zero internal impedance. $Y_G = \infty$, $r_1 = 1$ and consequently

$$\frac{E_2}{E_0} = e^{N\theta_{12}} \cdot \frac{1 - r_2}{1 - e^{N(\theta_{12} + \theta_{21})} r_2}\,. \tag{16}$$

This is the situation that usually prevails in the study of a feedback amplifier.

2. The source replaced by a constant current generator with an admittance Y_G across it in parallel. Then

$$E_2 = \frac{I_0}{Y_G + Y_T} \cdot e^{N\theta_{12}} \cdot \frac{1 - r_1 r_2}{1 - e^{N(\theta_{12} + \theta_{21})} r_1 r_2}\,, \tag{17}$$

where I_0 is the current output of the generator.

3. An interesting development of Case 2 arises if the constant current generator is a tube of the same type as are in the remainder of the amplifier.

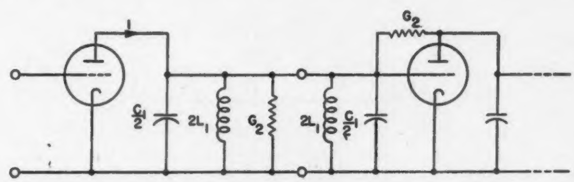

FIG. 5. *A special constant-current source arrangement.*

Let also a conductance G_2 be connected across the plate circuit. The circuit will be as shown in Fig. 5. It can be shown that

$$r_1 = e^{\theta_{12} + \theta_{21}}\,,$$

$$1 - r_1 r_2 = -(1 - r_2)(Y_T + y_{11})\frac{e^{\theta_{21}}}{y_{12}}\,.$$

This leads to

$$E = \frac{E_o e^{(N+1)\theta_{12}}(1 - r_2)}{\left(1 - \frac{G_2}{g_m}\right)\{1 - e^{(N+1)(\theta_{12} + \theta_{21})} r_2\}}\,. \tag{18}$$

Equation (18) will be observed to be the same as Equation (16) except for the constant factor $\frac{1}{1 - \frac{G_2}{g_m}}$ and the fact that N is replaced by $N + 1$. $1 - \frac{G_2}{g_m}$ is very close to unity in all practical cases, for example, in the present case it is 0.99. The result is, then, that the response of a feedback amplifier fed by a constant voltage generator is nearly unchanged by the removal of the first feedback resistor. The input admittance of the amplifier is, of course, considerably changed.

Calculation of Some Reflection Coefficients

The excellence of the response curve, judged from the viewpoint mentioned at the outset, i.e. the approach to the ideal amplifier, is determined mainly by the reflection coefficients r_1 and r_2. Examination of Equation (15) shows that the ideal response may be achieved by making $r_1 r_2 = 0$ and $\frac{Y_G}{Y_G + Y_T} = 1$ over the bandwidth of the amplifier. A terminating admittance for either end equal to the image admittance of the amplifier at all frequencies in the band cannot be made with finite networks. Considerable advantage is gained if the constants of the termination are chosen to make the reflection coefficient as small as possible. Results are particularly good if both r_1 and r_2 are small, provided the factor $\frac{Y_G}{Y_G + Y_T}$ remains reasonably constant.

The image admittance at mid-band is a pure conductance, which will be designated as G_0. From Equation (12) it is seen that

$$G_0 = \sqrt{g_m G_2} \, .$$

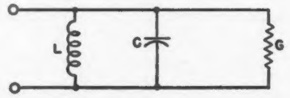

FIG. 6. *Simple termination admittance.*

The remainder of the section is devoted to numerical results on the reflection coefficients of a number of simple terminations. They are plotted in terms of $\frac{x}{x_0}$. Very approximately $\frac{x}{x_0} = 1$ corresponds to the band limits, and $\frac{x}{x_0} = 0$, as previously remarked, to the mid-band point. The magnitudes of the

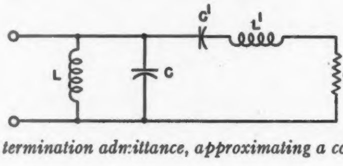

FIG. 7. *Improved termination admittance, approximating a constant-k half-section.*

coefficients are even functions, and their phases are odd functions, of x. This observation applies as well to the response curves that follow in the last part of the paper.

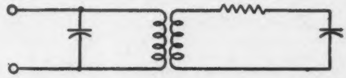

FIG. 8. *An admittance equivalent to that of Fig. 7.*

The results are shown in Figs. 9 to 11, both magnitudes and phases being given. Fig. 9 is for terminations 1, 2, and 3; Fig. 10 for terminations 4 and 5; and Fig. 11 for terminations 6 and 7.

The circuits of all the terminations studied are shown in Figs. 6 and 7. In Table I are given the capacities and conductance associated with each termination. In all cases

$$LC = L'C' = \frac{1}{\omega_0^2},$$

or, in other words, the circuits are all resonant at the mid-band frequency.

TABLE I

CONSTANTS OF THE TERMINAL IMPEDANCES

Termination	Network	C	C'	G
1	Fig. 6	$0.5\ C_1$	—	G_0
2	Fig. 6	$0.5\ C_1$	—	$0.5\ G_0$
3	Fig. 6	$0.5\ C_1$	—	$2\ G_0$
4	Fig. 6	$0.081\ C_1$	—	$0.611\ G_0$
5	Fig. 6	$0.080\ C_1$	—	$0.918\ G_0$
6	Fig. 7	$0.403\ C_1$	$\frac{2.67\ G_0^2}{\omega_0^2 C_1}$	G_0
7	Fig. 7	$0.386\ C_1$	$\frac{2.01\ G_0^2}{\omega_0^2 C_1}$	G_0

Terminations 1 to 3 are not recommended as good ones. The capacity is too large to allow the reflection coefficient to be reasonably small over much of the band. They are presented here because they arise frequently in practice.

The constants for terminations 4 and 5 have been selected to give zero reflection coefficient at $\frac{x}{x_0} = \pm 0.8$ and $\frac{x}{x_0} = \pm 0.4$, respectively. These two circuits give as good results as can be expected from such simple networks. It will be noted that practically no capacity is added to the end of the chain by either of these terminations.

In the case of terminations 6 and 7, there is an additional circuit providing another parameter that can be used to improve the terminal conditions. For these two terminations, the constants have been selected so that the reflection coefficient vanishes at three points in the band, $\frac{x}{x_0} = 0, \pm 0.4$, and $\frac{x}{x_0} = 0, \pm 0.8$, respectively.

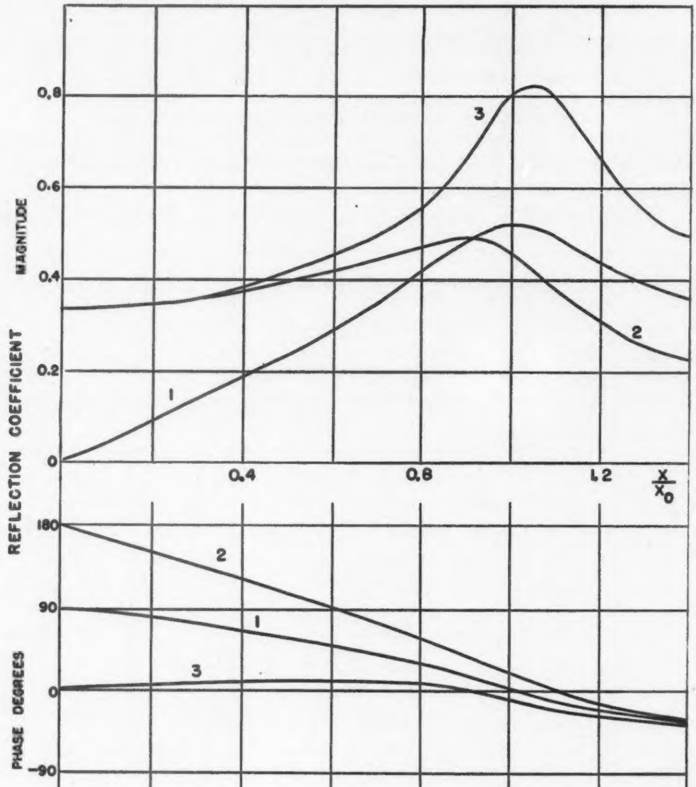

FIG. 9. *Reflection coefficients for terminations 1, 2, and 3.*

It may happen that the added elements of Fig. 7 are either too large or too small to be practical. In this case the equivalent circuit of Fig. 8 may be used with a transformation of impedance to secure circuit values of a reasonable size. These two terminations have approximately the constants of a constant-k half-section. It would be possible to make the conductance G

the natural terminating device; for example, the second detector. However, the conductance must have no appreciable capacity across it, and it is difficult to find a practical detector that satisfies this condition. Another arrangement is to connect the detector to the plate side of the network, but the detector in this case must introduce no load, i.e., it would have to be an infinite impedance detector or the equivalent.

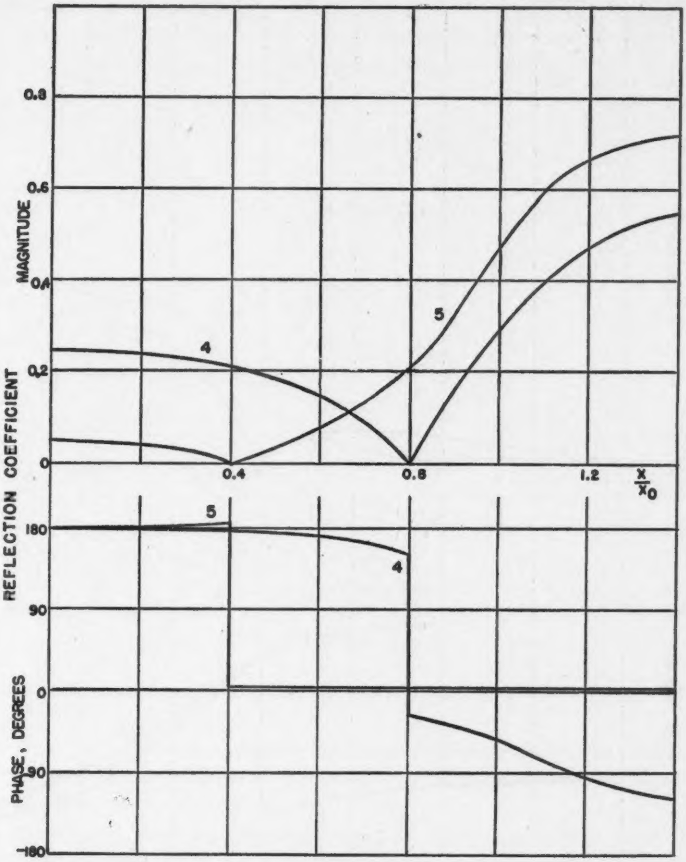

FIG. 10. *Reflection coefficients for terminations 4 and 5.*

Calculation of Some Response Curves

In this section are presented the results of numerical calculations of the response curves of finite chains of three, four, and five stages. They are

given in Figs. 12 to 21, the abscissae being $\frac{x}{x_0}$ as in the case of the reflection coefficients. Both the gain and phase shift are designated by the same letter in the diagrams, but the phase characteristic is distinguished by a prime on

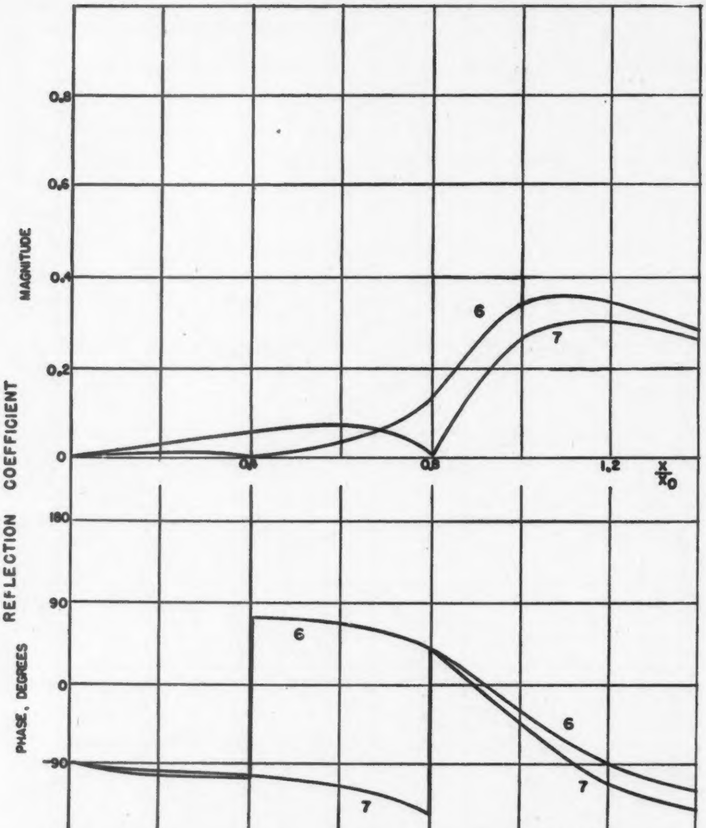

FIG. 11. *Reflection coefficients for terminations 6 and 7.*

the letter identifying it. Conforming with customary practice, the phase is shown as increasing from zero with increasing $\frac{x}{x_0}$, and the constant phase shift of 180°, obtained with amplifiers of odd numbers of stages, is ignored.

In Table II are listed the conditions for which each curve was calculated. While only the unprimed letters designating the gain curves are listed, the data apply also to the corresponding primed letters designating the phase responses. The numbers listed under "Source admittance" and "Terminal

TABLE II

TERMINAL CONDITIONS FOR RESPONSE CURVES

Figure	Curve	Number of stages	Type of source	Source admittance	Terminal admittance
12	*A*	3	Constant voltage	∞	Y_I
	B	4		∞	Y_I
	C	5		∞	Y_I
13	*A*	3	Constant voltage	∞	1
	B	3		∞	2
	C	3		∞	3
14	*A*	4	Constant voltage	∞	1
	B	4		∞	2
	C	4		∞	3
15	*A*	5	Constant voltage	∞	1
	B	5		∞	2
	C	5		∞	3
16	*A*	3	Constant voltage	∞	4
	B	3		∞	5
17	*A*	4	Constant voltage	∞	4
	B	4		∞	5
18	*A*	5	Constant voltage	∞	4
	B	5		∞	5
19	*A*	3	Constant voltage	1	1
	B	3		4	4
	C	3		5	5
20	*A*	3	Constant voltage	∞	6
	B	3		∞	7
21	*A*	3	Constant current	7	5
	B	4		7	5
	C	5		7	5

admittance" refer to the termination admittances described in Table I. Where ∞ and Y_I are given, it means an infinite admittance and one equal to the image admittance, respectively.

The curves of Fig. 12 are the ideal responses. The responses in Figs. 13 to 15 are quite unsatisfactory—with the possible exception of Curve *A* of Fig. 13. They are, however, the ones that will probably be first discovered in experimental work with feedback chains. The curves of Figs. 16 to 18 are better, but are probably still not good enough for most applications.

Fig. 19 illustrates the great improvement in response shape resulting from matching both input and output. This general conclusion also applies in cases where the number of stages is other than three. It will be noted that half of the over-all gain is lost in this arrangement.

In the circuits for Fig. 20, the output has been taken off the plate of the last tube and not off the load resistor, as was earlier suggested might be done.

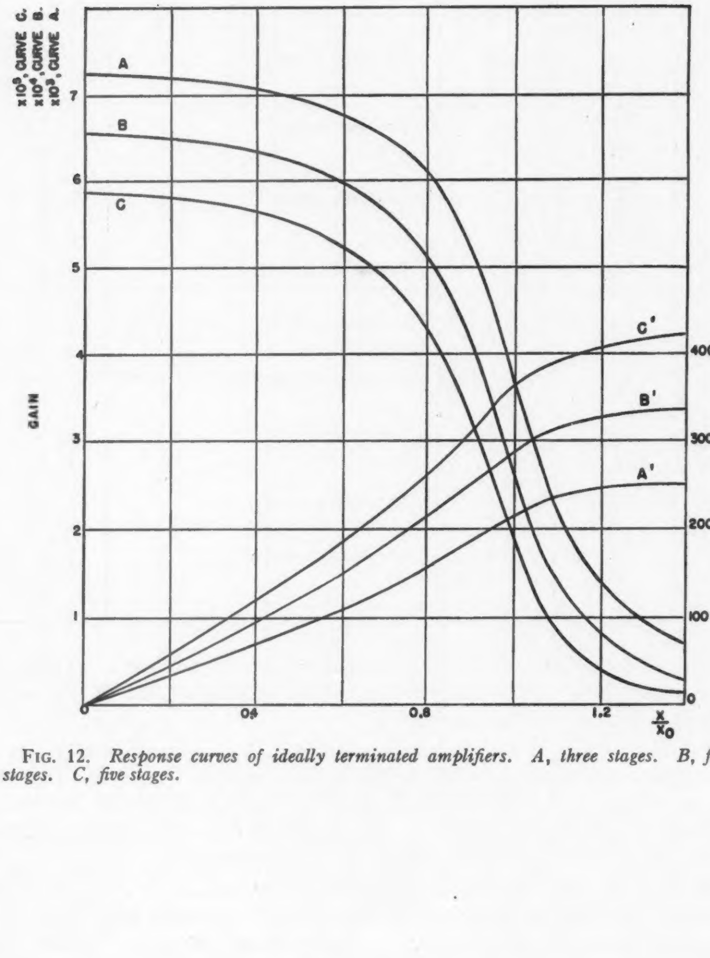

FIG. 12. *Response curves of ideally terminated amplifiers. A, three stages. B, four stages. C, five stages.*

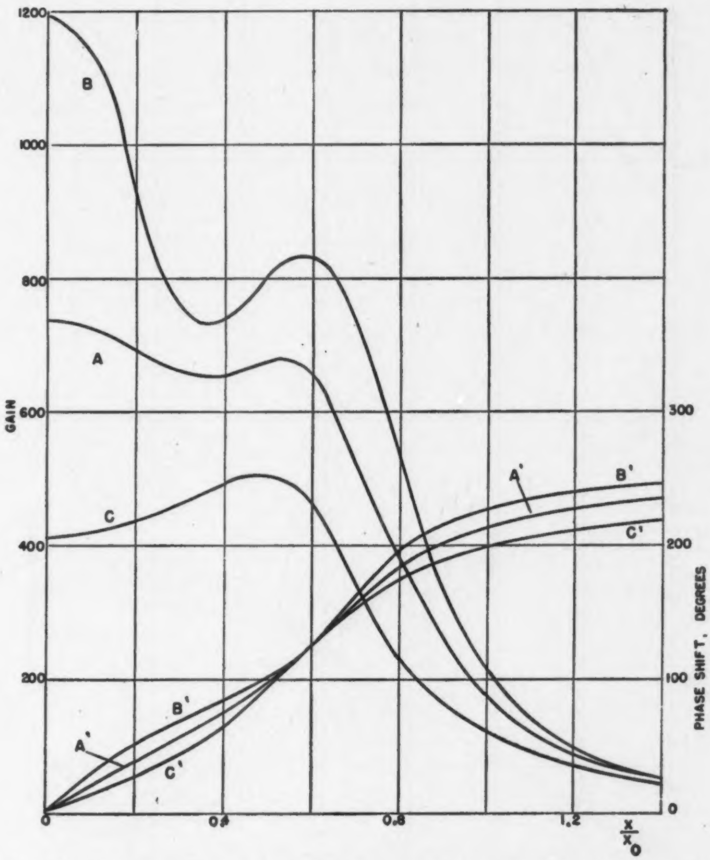

FIG. 13. *Response curves of three-stage amplifiers using: A, termination 1; B, termination 2; C, termination 3.*

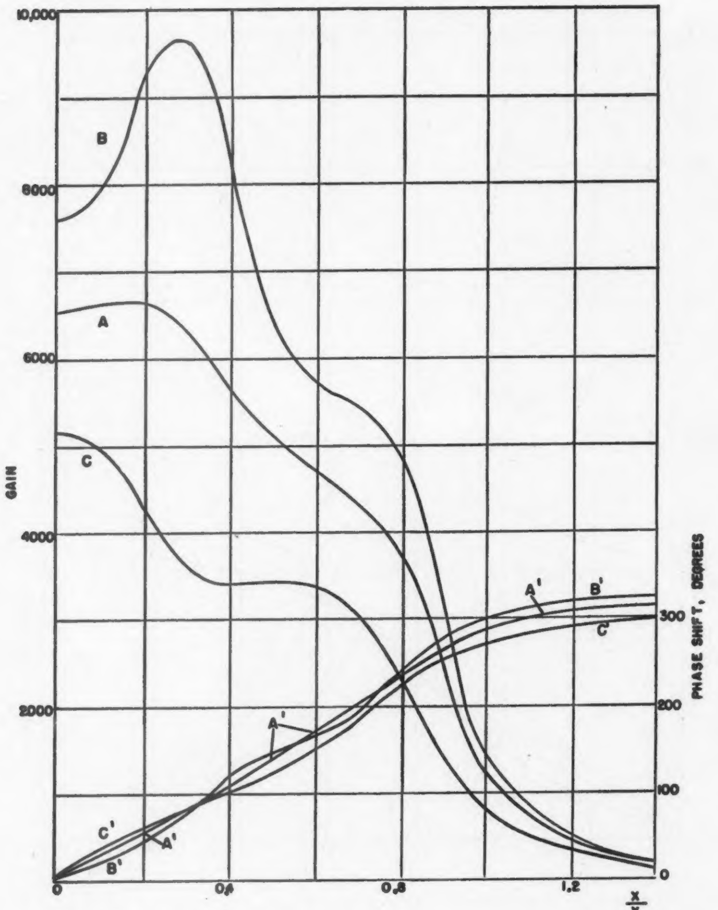

FIG. 14. *Response curves of four-stage amplifiers using: A, termination 1; B, termination 2; C, termination 3.*

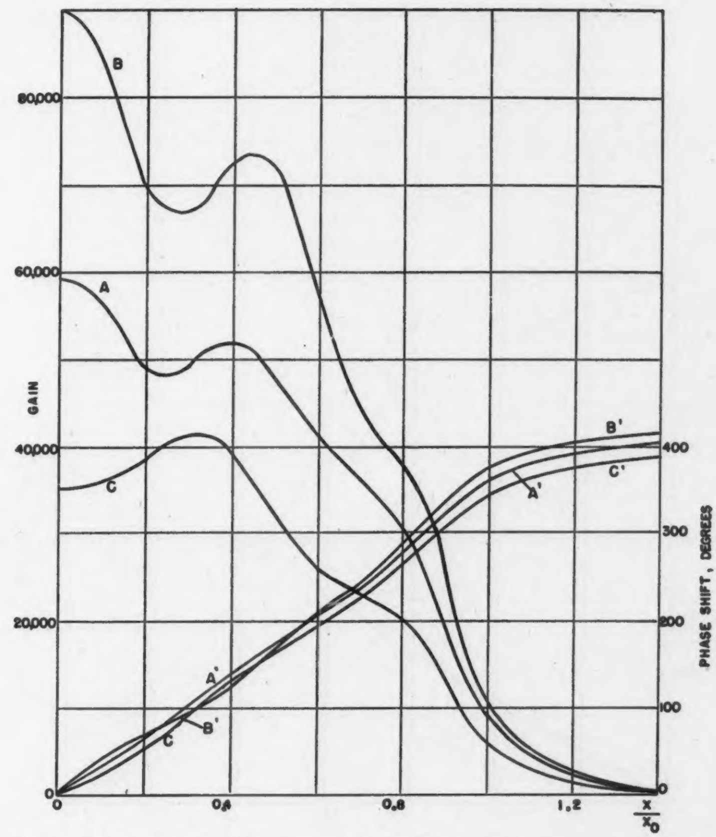

FIG. 15. *Response curves of five-stage amplifiers using: A, termination 1; B, termination 2; C, termination 3.*

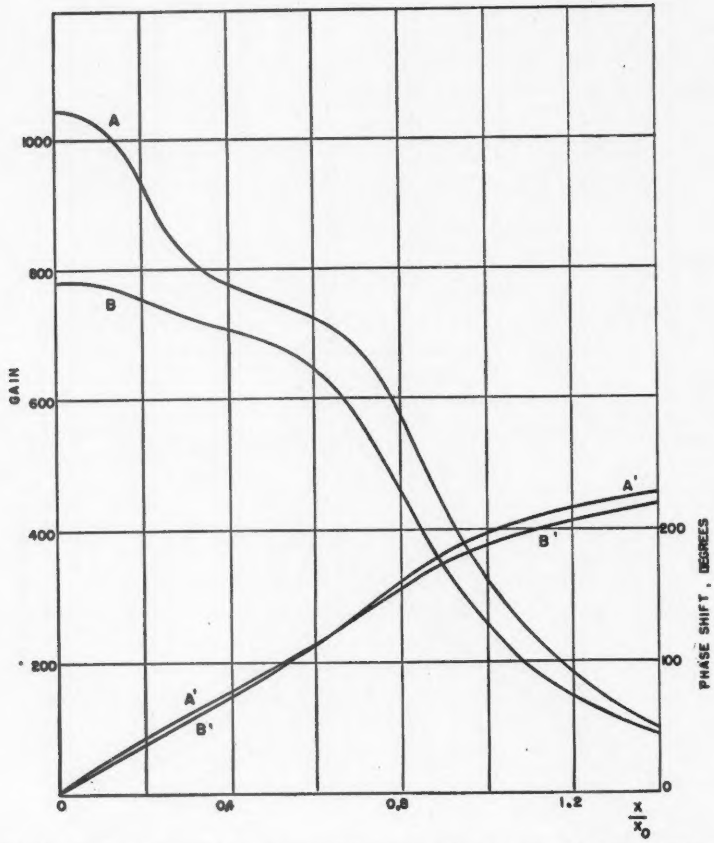

FIG. 16. *Response curves of three-stage amplifiers using: A, termination 4; B, termination 5.*

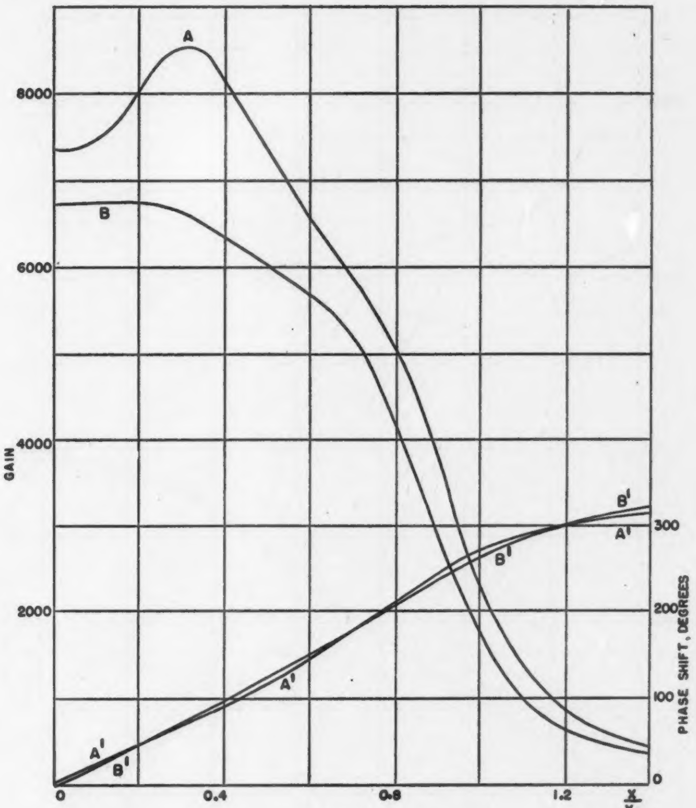

FIG. 17. *Response curves of four-stage amplifiers using: A, termination 4; B, termination 5.*

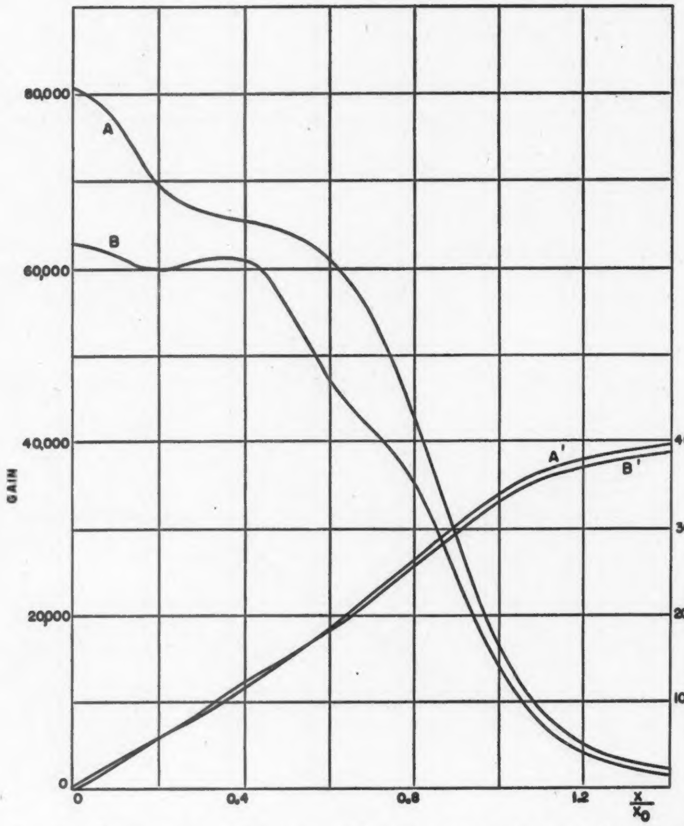

FIG. 18. *Response curves of five stage amplifiers using: A, termination 4; B, termination 5.*

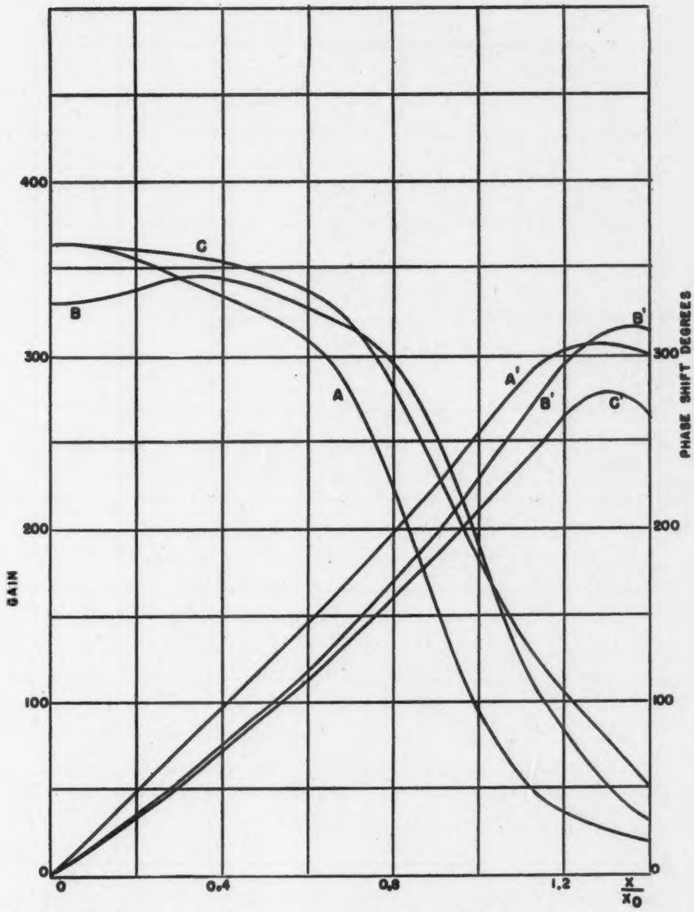

FIG. 19. *Response curves of three-stage amplifiers matched at both ends. A, termination 1; B, termination 4; C, termination 5.*

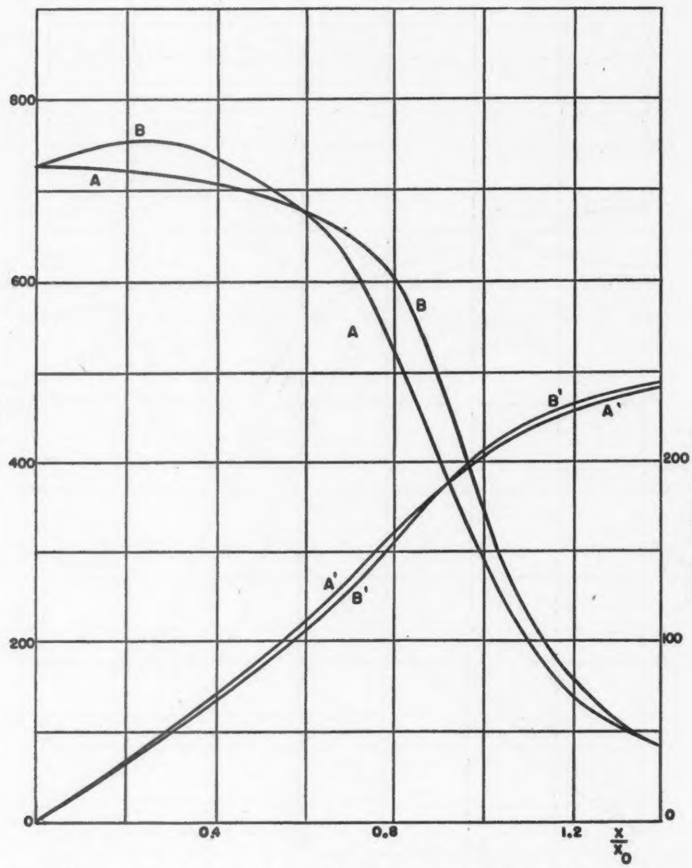

FIG. 20. *Response curves of three-stage amplifiers using: A, termination 6; B, termination 7.*

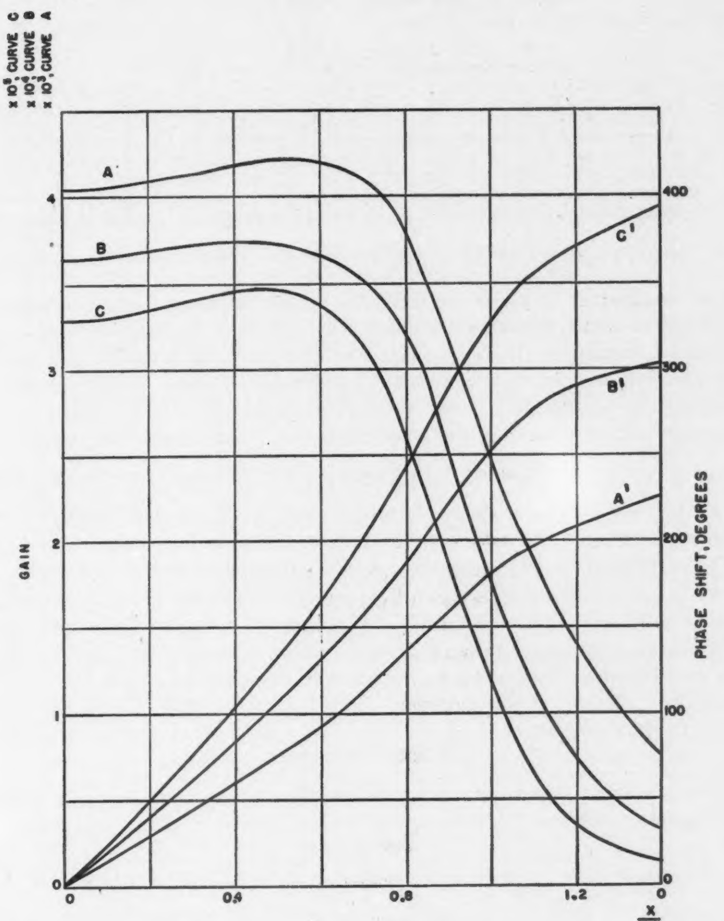

FIG. 21. *Response curves of amplifiers matched at both ends using terminations 5 and 7. A, three stages. B, four stages. C, five stages.*

Fig. 22 shows the complete three stage amplifier which gives rise to Curve *A* of Fig. 21. The others are precisely the same in their terminal arrangements. This arrangement makes $r_1 r_2$ zero at five points through the band. It has

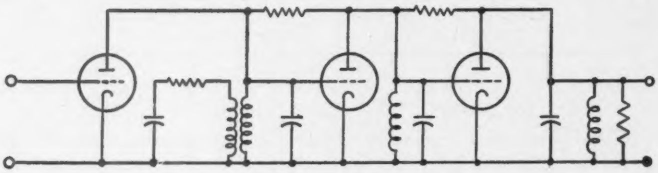

FIG. 22. *Circuit for Curve A of Fig. 21.*

been constructed to avoid the difficulty arising with the load mentioned in connection with terminations 6 and 7. It suffers, of course, from the disadvantage common to all systems matched at both ends, namely, that half of the gain is lost. In place of this a system is gained whose frequency characteristics are acceptable for any number of stages. The slight "ears" in the response, i.e., the rise near the band limits, is caused by the behaviour of the term $\frac{1}{Y_G + Y_T}$ of Equation (17). In an actual system, the direct loading that is inevitable across each tube would depress the response near the band limits and compensate somewhat this effect.

The curves show that large capacity is undesirable at the termination of a feedback chain. If this cannot be avoided then the final feedback unit can be modified so that the additional capacity is incorporated into it. This will change the image impedance at both ends and they will no longer be equal. By an increase of the feedback conductance, Y_{I1} can be readjusted to conform with the remainder of the amplifier. This procedure will entail the inevitable loss in gain.

Acknowledgment

The considerable labour of computing these curves has been carried out by Sgt. R. F. Pilgrim.

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